Individuals Round 1 States 2025

3 pts 1. Find the least common multiple (LCM) of 75 and 81.

Ans. ________________

4 pts 2. For what positive value of k will the circle $x^2 + y^2 - 6x + ky + 9 = 0$ have a radius of 5?

Ans. ________________

5 pts 3. A high-tension power line that is straight passes 2550 feet due north of a house and 1360 feet due east of the house. What is the closest the line gets to the house?

Ans. ______________________

Individuals Round 2 States 2025

3 pts 1. Find x, if $\frac{1}{x} + \frac{2}{x} + \frac{3}{x} = \frac{1}{4}$.

Ans. _________________

4 pts 2. If the decimal (base 10) number 2025 is written in the binary (base 2) system, how many of its 11 digits will be 1's?

Ans. _________________

5 pts 3. The 10^{th} term of a 50-term arithmetic sequence is 13 and the 41^{st} term is 18. Find the sum of all terms in the sequence.

Ans. ______________________

Individuals Round 3 States 2025

3 pts 1. Find the value of constant k so that the line $5x + ky + 17 = 0$ passes through the point (2, 5).

Ans. ________________

4 pts 2. Zippy enters a short triathlon that involves running for 4 km, swimming for 1 km, and biking for 8 km. She averages 6 m/s running, 2 m/s swimming and 15 m/s biking. m/s = meters per second. The whole triathlon takes her MM:SS, where MM and SS are non-negative for the number of minutes and the number of seconds and SS < 60. Find her total time and express it in MM:SS format.

Ans. __________________________

5 pts 3. From point P, the tangent of the angle from horizontal on the ground to the top of a flagpole is 13/10. From point Q, twenty feet further from the base of the flagpole, the tangent of the angle from the horizontal on the ground to the top of the flagpole is 13/14. Find the height of the flagpole in feet. Assume the ground in this problem is flat.

Ans. _____________________

Individuals Round 4 States 2025

3 pts 1. Find $58\frac{1}{3}$ % of 84.

Ans. _____________________

4 pts 2. Find M if, $\log_3 N = 2 + \frac{1}{\log_2 3}$ and $\log_M N = 1 - \frac{2}{\log_2 M}$.

Ans. _____________________

5 pts 3. A ball is thrown at a somewhat upward angle and its position is given by the parametric equations $x = 32t$ and $y = 6 + 48t - 16\,t^2$, where t = the elapsed time in seconds since the ball was thrown, x = the horizontal distance in feet the ball has traveled at time t, and y = the height of the ball in feet at time t. Find the location of the ball when it reaches its zenith (highest point). Write your answer as an (x, y) ordered pair.

Ans. _____________________

Individuals Round 5 States 2025

3 pts 1. Three lines are drawn in a plane and no two are parallel. The lines can be drawn in any way where all acute angles have measures of *m* degrees. Find the sum of *m* and *n*, where *n* is the maximum number *m* degree angles that can be formed.

Ans. ____________________

4 pts 2. Find the value of the expression below if $\theta = 30^o$.

$$\sqrt{1 - \frac{1}{cos^2\theta + tan^2\theta + sin^2\theta}}$$

Ans. ____________________

5 pts 3. How many of the 21 natural numbers in the set {110, 111, 112, . . . , 129, 130} are composite numbers?

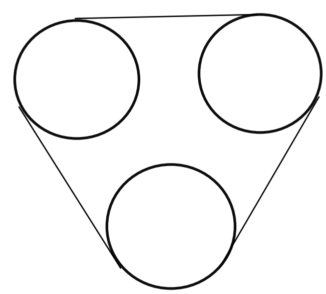
Ans. ____________________

Individuals Round 6 States 2025

3 pts 1. Define 2, 3, 5, 6, 8, 9 and 0 as "curly digits", since most people write them using curves. How many of the 900 3-digit whole numbers can be formed using only "curly digits"?

Ans. ____________________

4 pts 2. As shown, three pulleys, each of radius of *r*, are mounted in a plane so that their centers are vertices of an equilateral triangle. A fan belt is then installed over the pulleys. If each pulley is at a distance *r* from the other two pulleys, then the length of the belt can be expressed as *kr*, where *k* is a constant. Find the value of *k*.



Ans. ____________________

5 pts 3. Three beakers are placed on a rack, the left beaker (L), the middle beaker (M) and the right beaker (R). Beaker L has a capacity of 5 oz. and is initially full of water. Beakers M and R have capacities of 4 oz. and 2 oz., respectively, and are initially empty. Using from 1 to 3 *pour* operations, how many possibilities are there for water left in beaker L after the *pours*? A *pour* operation consists of pouring all the contents of one beaker into another. If the contents of the pouring beaker is greater than the remaining capacity of the receiving beaker, the excess remains in the pouring beaker.

Ans. ____________________

Team 1 States 2025

4 pts 1. A math teacher has several polygonal cutouts, a triangle, a pentagon, a nonagon, a dodecagon and an icosagon. Find the sum of the number of sides of all the cutouts.

(1) Ans. ____________4 pts

4 pts 2. Convert each of the five expressions below into an integer and give the largest integer as your answer: 2^9 $17 \cdot 31$ $31^2 - 21^2$ 8^3 23^2

(2) Ans. ______________ 4 pts

6 pts 3. Elmer has a goal to average $3.\overline{7}$ or lower per hole in a 9-hole golf tournament round. After 7 holes, his average is $3.\overline{285714}$ shots per hole. What is the maximum total number of shots he can take on the final two holes and still achieve his goal?

(3) Ans. ____________ 6 pts

6 pts 4. Find the sum of the entries that result from $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$ when $\theta = 30^{\circ}$, $x = 10$ and $y = 2$.

(4) Ans. _____________ 6 pts

6 pts 5. The solutions for the following equation (using our simplification rules) are in the form $\frac{a \pm b}{c}$. $\frac{2x-1}{x^2+x-6} + \frac{4x+3}{x^2+7x+12} = \frac{3x-1}{x^2+2x-8}$ Find the value of $a^2 + b^2 + c^2$.

(5) Ans. _____________ 6 pts

8 pts 6. 5 points are plotted in the *xy*-plane. Point P at (4, 3), Q at (13, 10), R at (16, -1), S at (7, -8) and T at (-6, -2). Find the smallest distance between any two of these points.

(6) Ans. ______________ 8 pts

8 pts 7. Simplify completely: $\left(\frac{\sqrt[5]{m^{15}n^{75}} \cdot \sqrt[3]{m^{27}n^{33}}}{\sqrt[4]{m^{24}n^{44}} \cdot \sqrt[6]{m^{54}n^{72}}}\right)^{-\frac{2}{3}}$. Give answer with positive exponents.

(7) Ans. _____________ 8 pts

8 pts 8. Players A, B and C each start with the same positive integer, N, of spirit beads and do a process involving 8 passes. In the first four passes, each passes one-half of the beads he/she then has, first A to B, then B to C, then C to B and then B to A. The last four passes are identical to the first four, except that each player passes one-third of the beads he/she has. If N is the minimum so that all passes involve a whole number of beads, how many beads does B have at the end?

(8) Ans. _____________ 8 pts

Team 2 States 2025

4 pts 1. The number 3 is a root of $x^3 - 13x^2 + 80x - 150$ and the other two roots are complex numbers. Find the product of these two complex numbers.

(1) Ans. _______________ **4 pts**

4 pts 2. Points P and Q are located in 3-dimensional space as follows:
P(5, 1, 4) and Q(-7, 10, -4). Find the distance from P to Q. **(2) Ans.** ________________ **4 pts**

6 pts 3. Find the smallest 3-digit whole number greater than 113 that is prime.

(3) Ans. ________________ **6 pts**

6 pts 4. Find the sum of the arithmetic series:

95 + 87 + 79 + . . . + (-89) + (-97). **(4) Ans.** ________________ **6 pts**

6 pts 5. Suppose N is a positive integer less than 100 with the following two properties:
(1) N is a prime number, (2) $N + 2$ is also a prime number.

Find the sum of all possible values of N. **(5) Ans.** _________________ **6 pts**

8 pts 6. Grace and Sophie flip a fair coin every evening to determine which of them will get to spend an hour playing with and grooming their cat, Muggles, that night. The probability that Grace will get exactly 4 Muggles nights in the next seven days is $\frac{m}{n}$, a fully simplified fraction. Find $m + n$.

(6) Ans. _________________ **8 pts**

8 pts 7. To encrypt a word, an algorithm uses four steps as follows:

(1) Turn each letter's rank in the alphabet. (A = 1, B = 2, … , Z = 26)

(2) Write the sequence of numbers backwards.

(3) Subtract each number from 27.

(4) Convert each number into its corresponding letter (1 = A, 2 = B, 26 = Z).

Decipher: VVWZPXRSX

(7) Ans. ____________________ **8 pts**

8 pts 8. The solutions for $\begin{vmatrix} x-1 & x+3 & 5 \\ 2 & x-5 & 3 \\ x & 2 & x-3 \end{vmatrix} = 89$ are $x = d$ and $x = \frac{a \pm \sqrt{b}}{c}$, where a, b and c are relatively prime. Find the value of $a + b + c + d$.

(8) Ans. ____________________ **8 pts**

Seat A Blue Relay States 2025

Find the only positive integer satisfying $3(8-x)-12\left(\frac{x^2}{12}-\frac{x}{4}-10\right)=0$

Pass back: 4A A = Your answer

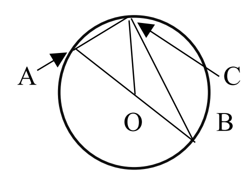
Seat B Blue Relay States 2025

Jerry can eat a whoopie pie every 40 seconds. Willy can eat one every 48 seconds. How many total whoopie pies can they eat in 4 minutes?

Pass back: X – 4B B = Your answer X = The number you will receive

Seat C Blue Relay States 2025

$\overline{AB}$ is a chord in the circle with center O and O is on $\overline{AB}$. Point C is on the circle and m∠AOC = 29º. Find the measure of angle ACB in degrees.



Pass back: $\frac{2C}{X}$ C = Your answer X = The number you will receive

Seat D Blue Relay States 2025

Find the product of the two solutions of $5x^2+25x-180=0$.

Pass back: $\sqrt{\frac{5D}{-X}}$ D = Your answer X = The number you will receive

Seat E Blue Relay States 2025

A gondola has 6 seats arranged into two rows of 3 seats each, with each row facing the other. In how many ways can 6 people be seated in the gondola, if April and Ben insist on sitting next to each other, Chris and Dottie refuse to sit either next to each other or directly across from each other, and Earl and Fritz don't care where they sit?

Pass in: $\frac{E}{X^2}$ E = Your answer X = The number you will receive

Seat A Green Relay States 2025

Find the only positive integer satisfying $3(11 - x) - 4\left(\frac{x^2}{4} - \frac{3x}{4} - 22\right) = 0$.

Pass back: 4A A = Your answer

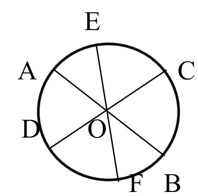
Seat B Green Relay States 2025

A cold water faucet can fill a quart container in 12 seconds. A hot water faucet, in a different sink, can fill a quart container in 16 seconds. Using both faucets, how many quarts can be filled in 4 minutes?

Pass back: X – B B = Your answer X = The number you will receive

Seat C Green Relay States 2025

Chords AB, CD and EF all pass through O, the center of the circle shown. If m $\widehat{BC}$ = 75° and m $\widehat{AE}$ = 40°, find m $\widehat{DF}$ in degrees.



Pass back: C – 3X C = Your answer X = The number you will receive

Seat D Green Relay States 2025

Find the product of the two solutions of $4x^2 - 8x - 140 = 0$.

Pass back: D – X D = Your answer X = The number you will receive

Seat E Green Relay States 2025

A gondola has 8 seats arranged into two rows of 4 seats where each row faces the other. In how many ways can 8 different people be seated in the gondola, if April, Ben and Chris insist on sitting in 3 adjacent seats with Ben in the middle, Dottie insists on sitting in one of the end seats, and the four other people don't care where they sit?

Pass in: E + 7X E = Your answer X = The number you will receive

Seat A Pink Relay States 2025

For what value of k is $8^k = 1/4$?

Pass back: 12A A = Your answer

Seat B Pink Relay States 2025

Judi has received one 73, seven 80's, ten 85's, nine 90's, five 95's and four 100's on her Latin quizzes this semester. What is her Latin quiz average?

Pass back: B – 10X B = Your answer X = The number you will receive

Seat C Pink Relay States 2025

The vertices of a triangle are A (5, 8), B (2, 6) and C (-2, 12). Calculate the area of the triangle.

Pass back: 5C + X + 10 C = Your answer X = The number you will receive

Seat D Pink Relay States 2025

The expression $\frac{2x^5-16x^4+11x^3+144x^2-251x+30}{x^3-4x^2-11x+30}$ simplifies to an expression of the form $ax^2 + bx + c$, where a, b and c are integers. Find the product abc.

Pass back: X + 15D D = Your answer X = The number you will receive

Seat E Pink Relay States 2025

$f(x) = \frac{x+3}{x-2}$. If $f(f(f(x))) = \frac{ax+b}{cx+d}$, where c > 0 and $x \neq 2$ or 7, find the value of $ab - cd$.

Pass in: EX E = Your answer X = The number you will receive

Seat A Yellow Relay States 2025

For what value of k is $8^k = 1/16$?

Pass back: 24A A = Your answer

Seat B Yellow Relay States 2025

Burt has received one 60, eight 80's, nine 85's, nine 90's, six 95's and two 100's on his French quizzes this semester. What is his French quiz average?

Pass back: B – 3X B = Your answer X = The number you will receive.

Seat C Yellow Relay States 2025

The vertices of a triangle are A (4,3), B (8, 13) and C (9, 1). Find the area of the triangle.

Pass back: $7\left(\frac{X}{3} - 2C\right)$ C = Your answer X = The number you will receive

Seat D Yellow Relay States 2025

The expression $\frac{3x^5-22x^4+24x^3+83x^2-144x+20}{x^3-5x^2-4x+20}$ simplifies to an expression of the form $ax^2 + bx + c$, where a, b and c are integers. Find the product abc.

Pass back: X – 3D D = Your answer X = The number you will receive

Seat E Yellow Relay States 2025

$f(x) = \frac{x-2}{x+1}$. If $f(f(f(x))) = \frac{ax+b}{cx+d}$, where c > 0 and $x \neq -\frac{1}{2}$ or -1, find $abcd$.

Pass in: X – 8E E = Your answer X = The number you will receive

Solution Sheet Individuals Round 1 States 2025

1. $75 = 3\cdot5\cdot5$, $81 = 3\cdot3\cdot3\cdot3$. LCM $= 3^4\cdot5^2 = 2025$. **Ans. 2025**

2. $x^2 + y^2 - 6x + ky + 9 = 0 \Rightarrow (x^2 - 6x + 9) + (y^2 + ky + \frac{k^2}{4}) = -9 + 9 + \frac{k^2}{4} = 25 \Rightarrow k^2 = 100$.
Since k is positive, then $k = 10$. **Ans. 10**

3. The triangle with vertex at the house and the other two vertices on the power line makes a right triangle, with the house at the right angle. The ratio of the legs $= \frac{1360}{2550} = \frac{136}{255} = \frac{4\cdot34}{5\cdot51} = \frac{4\cdot2\cdot17}{5\cdot3\cdot17} = \frac{8}{15}$. Voila! An 8-15-17 right triangle, with a multiple of 170. The altitude to the hypotenuse $= \frac{ab}{c} = \frac{8\cdot15}{17}$. $170\left(\frac{8\cdot15}{17}\right) = 1200$. **Ans. 1200**

Individuals Round 2

1. $\frac{1}{x} + \frac{2}{x} + \frac{3}{x} = \frac{1}{4} \Rightarrow \frac{6}{x} = \frac{1}{4} \Rightarrow x = 24$. **Ans. 24**

2. 2025/2 = 1012R1, 1012/2 = 506R0, 506/2 = 253R0, 253/2 = 126R1, 126/2 = 63R0, 63/2 = 31R1, 31/2 = 15R1, 15/2 = 7R1, 7/2 = 3R1, 3/2 = 1R1, 1/2 = 0R1. This makes 8 ones. Reading backwards: $11, 111, 101, 001_2 = 2025_{10}$. **Ans. 8**

3. Note that the 10^{th} term is the tenth term from the end and the 41st term is the tenth term from the other side, so the average of the terms is $\frac{13+18}{2} = \frac{31}{2}$. $50\left(\frac{31}{2}\right) = 775$. Using normal procedures: $13 = a + 9d$ and $18 = a + 40d$. Subtracting, $d = \frac{5}{31}$ and $a = \frac{358}{31}$. 50^{th} term:
$L = \frac{358}{31} + 49\left(\frac{5}{31}\right) = \frac{358+245}{31} = \frac{603}{31}$. Sum $= 50\left(\frac{\frac{358}{31}+\frac{603}{31}}{2}\right) = 25\left(\frac{961}{31}\right) = 25(31) = 775$. **Ans. 775**

Individuals Round 3

1. $5x + ky + 17 = 0 \Rightarrow 5(2) + k(5) = -17 \Rightarrow 5k = -27, k = -27/5$. **Ans. -27/5**

2. Converting to meters: $4000/6 = 666\frac{2}{3}$ sec for running; $1000/2 = 500$ sec for swimming; $8000/15 = 533\frac{1}{3}$ sec for biking. Total 1700 sec. 1700/60 = 28min 20 sec. **Ans. 28:20**

3. Label the height of the flagpole h and the distance on the ground from point P to the base of the flagpole x. Since the flagpole makes a 90° angle with the ground, then (1): $\frac{h}{x} = \frac{13}{10}$ and (2): $\frac{h}{20+x} = \frac{13}{14} \Rightarrow$ In (1): $10h = 13x$. In (2): $14h = 260 + 13x$. $4h = 260$ and $h = 65$. **Ans. 65**

Individuals Round 4

1. $58\frac{1}{3}\ \% = \frac{58\frac{1}{3}}{100} = \frac{175}{300} = \frac{7}{12}$. $\frac{7}{12}(84) = 7(7) = 49$. **Ans. 49**

2. In the first equation: $\log_3 N = 2 + \frac{1}{\log_2 3}$ ➔ $\log_3 N = \log_3 9 + \log_3 2$ ➔ $\log_3 N = \log_3 18$, so $N = 18$. In the second equation: $\log_M N = 1 - \frac{2}{\log_2 M}$ ➔ $\log_M 18 = \log_M M - 2\log_M 2$ ➔ $18 = \frac{M}{4}$. So $M = 72$. **Ans. 72**

3. Observe that the equation is a parabola which opens downward. We need the vertex of the parabola, which at that time will be zenith: $y - 6 = -16(t^2 - 3t + \frac{9}{4}) + 36$ ➔ $y - 42 = -16(t - \frac{3}{2})^2$. $t = 3/2$. So $x = 32(\frac{3}{2}) = 48$. **Ans. (48, 42)**

Individuals Round 5

1. The three lines could intersect to form an equilateral triangle in which all acute angles formed are 60° and the vertical angles form 3 more 60° angles. Total of 6. If two of the lines meet at 90°, and the other intersects to form 45° angles, there are 4 of them. If the lines intersect at the same point and all angles are 60°, there are 6 of these. So greatest number 60° angles is 6. m = 60 and n = 6. Sum = 60 + 6 = 66. **Ans. 66**

2. $\cos^2\theta + \sin^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$. Thus $\frac{1}{sec^2\theta} = \cos^2\theta$ and $1 - \cos^2\theta = \sin^2\theta$. Therefore $\sqrt{sin^2\theta} = \sin\theta$ and $\sin 30° = 1/2$. **Ans. 1/2**

3. The 11 even numbers are all composite. 111, 117, 123, 129 are all multiples of 3; 115, 125 are multiples of 5. 119 = 7(19), $121 = 11^2$. 113 and 127 are prime. 11 + 4 + 2 + 2 = **Ans. 19**

Individuals Round 6

1. The first digit can be 2, 3, 5, 6, 8 and 9. The other two digits can be all these plus 0. So there are 6·7·7 = 294 3-digit numbers. **Ans. 294**

2. Each arc must be 120°. All form a circumference or $2\pi r$. The three segments from point of tangency of one pulley to another is each 3r. Total is $9r + 2\pi r$. $k = 9 + 2\pi$. **Ans. 9 + 2π**

3 Contents left in beaker L: 0 L → M (1-4-0), L➔R , (0-4-1); 1 L → M , (1-4-0); 2 L ➔R(3-0-2), L➔M, (0-3-2), R➔L (2-3-0), 3 L➔R, (3-0-2); 4 L➔M, (1-4-0), L ➔R, (0-4-1), M➔L, (4-0-1); 5 L➔M, (1-4-0), M➔L, (5-0-0) **Ans. 6**

Team Round 1

1. 3 + 5 + 9 + 12 + 20 = 49 **Ans. 49**

2. $2^9 = 512$; 17(37) = 527; $31^2 - 21^2 = (52)(10) = 520$; $8^3 = 2^9 = 512$, $23^2 = 529$. **Ans. 529**

3. $3.\overline{285714} = 3\frac{2}{7}$. So it took 23 shots on the first 7 holes. To make an average of $3.\overline{7}$, which is $3\frac{7}{9}$ is $\frac{34}{9}$, he needs to finish with 11 shots maximum. **Ans. 11**

4. $\begin{bmatrix}\cos 30 & -\sin 30\\ \sin 30 & \cos 30\end{bmatrix}\cdot\begin{bmatrix}10\\ 2\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\cdot\begin{bmatrix}10\\ 2\end{bmatrix}=\begin{bmatrix}5\sqrt{3}-1\\ 5+\sqrt{3}\end{bmatrix}$. Sum = $6\sqrt{3}+4$. **Ans. $6\sqrt{3}+4$**

5. $\frac{2x-1}{x^2+x-6}+\frac{4x+3}{x^2+7x+12}=\frac{3x-1}{x^2+2x-8}$ ➔ $\frac{2x-1}{(x-2)(x+3)}+\frac{4x+3}{(x+3)(x+4)}=\frac{3x-1}{(x-2)(x+4)}$ ➔
$(2x - 1)(x + 4) + (4x + 3)(x – 2) = (3x - 1)(x + 3)$ ➔ $2x^2 + 7x - 4 + 4x^2 – 5x - 6 = 3x^2 + 8x – 3$.
$6x^2 + 2x - 10 = 3x^2 + 8x – 3$ ➔ $3x^2 – 6x - 7 = 0$. $x=\frac{6\pm\sqrt{36-4(-21)}}{6}=\frac{6\pm\sqrt{120}}{6}=\frac{6\pm 2\sqrt{30}}{6}=$
$\frac{3\pm\sqrt{30}}{3}=\frac{a\pm b}{c}$. $a^2 + b^2 + c^2 = 9 + 30 + 9 = 48$. **Ans. 48**

6. P(4, 3), Q(13, 10), R(16, -1), S(7, -8) and T(-6, -2). $(PQ)^2 = 9^2 + 7^2 = 130$; $(PR)^2 = 12^2 + 4^2 = 160$; $(PS)^2 = 3^2 + 11^2 = 130$; $(PT)^2 = 10^2 + 5^2 = 125$; $(QR)^2 = 3^2 + 11^2 = 130$; $(QS)^2 = 6^2 + 18^2 = 360$; $(QT)^2 = 19^2 + 12^2 = 505$; $(RS)^2 = 505$; $(RT)^2 = 22^2 + 1^2 = 485$; $(ST)^2 = 13^2 + 6^2 = 205$. The smallest of these numbers is 125. $\sqrt{125} = 5\sqrt{5}$. **Ans. $5\sqrt{5}$**

7. $\left(\frac{\sqrt[5]{m^{15}n^{75}}.\sqrt[3]{m^{27}\cdot n^{33}}}{\sqrt[4]{m^{24}n^{44}}.\sqrt[6]{m^{54}n^{72}}}\right)^{-\frac{2}{3}}$ ➔ $\left(\frac{m^3n^{15}m^9n^{11}}{m^6n^{11}m^9n^{12}}\right)^{-\frac{2}{3}}=\left(\frac{m^{12}n^{26}}{m^{15}n^{23}}\right)^{-\frac{2}{3}}=\frac{n^3}{m^3}^{-\frac{2}{3}}=\frac{n^{-2}}{m^{-2}}=$
$\frac{m^2}{n^2}$. **Ans. $\frac{m^2}{n^2}$**

8. Construct a table for each player in the pass, making sure each player in each column has the same denominator and that each denominator can be divisible by a 2 or 3.

Pass	Initial	1	2	3	4	5	6	7	8
Player A	1/1	1/2	2/4	4/8	21/16	14/16	42/48	126/144	560/432
Player B	1/1	3/2	3/4	13/8	13/16	20/16	40/48	182/144	364/432
Player C	1/1	2/2	7/4	7/8	14/16	14/16	62/48	124/144	372/432

Ans. 364

Team Round 2

1. Dividing the trinomial by x – 3, will produce the constant 50, the product of the roots of the quadratic. **Ans. 50**

2. The distance between two points in the three-dimensional plane is basically the Pythagorian Theorem $\sqrt{a^2+b^2+c^2}=\sqrt{12^2+\ 9^2+8^2}=\sqrt{144+81+64}=\sqrt{289}=17$. **Ans. 17**

3. Any even number, and numbers ending in 5 are out. That leaves 117,div by 3; 119, div by 7; 121, div by 11, 123,div by 3; 127, not div by 3, 7 or 11, is as high as need to check.

Ans. 127

4. -97 = 95 + (n – 1)(-8) ➔ -192 = -8n + 8, 8n = 200, so n = 25, number of terms. Sum = 25(-2/2)= - 25. **Ans. – 25**

5. There are 8 values for N that meet the requirements: 3, 5, 11, 17, 29, 41, 59, 71. The sum is 8 + 28 + 70 + 130 = 236. **Ans. 236**

6. Since each flip will be independent of each flip. The probability then will be 4 "good" results out of the 7 flips is $\binom{7}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3 = \frac{7\cdot6\cdot5\cdot4!}{3\cdot2\cdot4!}\cdot\frac{1}{2^7} = \frac{35}{128} = \frac{m}{n}$. m +n = 163. **Ans. 163**

7. Initial: V – V – W – Z – P – X – R – S – W

Then : 22 – 22 – 23 – 26 – 16 – 24 – 18 – 19 – 24

Then : 24 – 19 – 18 – 24 – 16 – 26 – 23 – 22 – 22

Then : 3 – 8 – 9 – 3 – 11 – 1 – 4 – 5 – 5

Then : C – H – I – C – K – A – D – E – E (our state bird)

Ans. CHICKADEE

8. $\begin{vmatrix} x-1 & x+3 & 5 \\ 2 & x-5 & 3 \\ x & 2 & x-3 \end{vmatrix} = 89$ ➔

(x – 1)(x^2 – 8x + 15) + 3x(x + 3) + 20 – 5(x^2 – 5x) – 6(x – 1) – 2(x^2 – 9) = 89 ➔
x^3 – 9x^2 + 23x – 15 + 3x^2 + 9x + 20 – 5x^2 + 25x – 6x + 6 – 2x^2 + 18 = 89 ➔
x^3 – 13x^2 + 51x – 60 = 0 ➔ (x – 4)(x^2 – 9x + 15 = 0. So x = 4, or x = $\frac{9 \pm \sqrt{81-4(15)}}{2} = \frac{9 \pm \sqrt{21}}{2} = \frac{a \pm \sqrt{b}}{c}$. So a = 9, b = 21, c = 2 and d = 4. $a + b + c + d$ = 9 + 21 + 2 + 4 = 36. **Ans. 36**

Seat A Blue Relay

$3(8 - x) - 12\left(\frac{x^2}{12} - \frac{x}{4} - 10\right) = 0$ ➔ 24 – 3x – x^2 + 3x + 120 = 0 ➔ 144 = x^2, x = 12.
Pass: 4A , 4(12) = 48. **A = 12, Pass 48**

Seat B Blue Relay

4 min = 240 sec. 240/40 = 6 by Jerry. 240/48 = 5 by Willy. Total 11. Pass: X – 4B, 48 – 4(11) = 4. **B = 11, Pass 4**

Seat C Blue Relay

Since chord AB passes through the center, angle ACB must be a right angle having 90° measure. Pass: 2C/X = 180/4 = 45. **C = 90, Pass 45**

Seat D Blue Relay

By Vieta's Formula, the product is c/a: (-180/5) = -36. Pass: $\sqrt{\frac{5D}{-X}} = \sqrt{\frac{5(-36)}{-(45)}} = 2$.

D = - 36, Pass 2

Seat E Blue Relay

There are 4 pairs of adjacent seats for A and B and two orders in which to assign them to the two seats. There remain 3 ways to pick seats for C and D and two orders to assign them to the two seats. Finally, there are 2 ways to assign E and F to the remaining seats. $4\cdot2\cdot3\cdot2\cdot2 = 96$.
Pass: $E/X^2 = 96/2^2 = 24$. **E = 96, Pass 24**

Seat A Green Relay

$3(11 - x) - 4\left(\frac{x^2}{4} - \frac{3x}{4} - 22\right) = 0 \Rightarrow 33 - 3x - x^2 + 3x + 88 = 0 \Rightarrow x^2 = 121, x = 11$.
Pass: 4A, 4(11) = 44. **A = 11, Pass 44**

Seat B Green Relay

4 min = 240 sec. 240/12 = 20. 240/16 = 15. Total = 35. Pass: X – B, 44 – 35 = 9.

B = 35, Pass 9

Seat C Green Relay

If $m\widehat{AE} = 40$, then $m\widehat{PB} = 40$ and $m\widehat{FC} = 115$. If $m\widehat{DFC} = 180$, then $m\widehat{DF} = 65$. Pass: C – 3X, 65 – 3(9) = 38. **C = 65, Pass 38**

Seat D Green Relay

The product of the roots is -140/4 = -35. Pass: D – X, -35 – 38 = -73. **D = -35, Pass -73**

Seat E Green Relay

There are 4 ways to pick the seats for A, B and C and then 2 ways to assign them to their particular seats. At that point, there will be 3 available seats for D. Once those have been assigned, there will be 4! ways to assign the remaining people to the last four seats.
$4\cdot2\cdot3\cdot24 = 576$. Pass: E + 7X, 576 + 7(-73) = 576 - 511 = 65. **E = 576, Pass 65**

Seat A Pink Relay

$8^k = 1/4 \Rightarrow 2^{3k} = 2^{-2}$. 3k = -2, k = -2/3. Pass: 12A, 12(-2/3) = -8. **A = -2/3, Pass -8**

Seat B Pink Relay

$\frac{73+560+850+810+475+400}{36} = \frac{3168}{36} = 88$. Pass: B – 10X, 88 – 10(-8) = 168. **B = 88, Pass 168**

Seat C Pink Relay

A(5, 8), B(2, 6), C(-2, 12). Slope of $\overline{AB} = \frac{8-6}{5-2} = \frac{2}{3}$. Slope of $\overline{BC} = \frac{6-12}{2-(-2)} = \frac{-6}{4} = \frac{-3}{2}$. The slopes are negative reciprocals, so $\overline{AB} \perp \overline{BC}$, and B is the vertex of the right angle. AB = $\sqrt{2^2+3^2} = \sqrt{13}$. BC = $\sqrt{(-6)^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$. Area = $\frac{1}{2}\left(2\sqrt{13}\cdot\sqrt{13}\right) = 13$ or using the shoelace formula (aka surveyor's formula), list the vertex points in counterclockwise order around the triangle and repeat the first point at the end. The area is the sum of the down-right products less the sum of the up-right products all over 2: (30 + 24 – 16 – 60 + 12 – 16) /2 = - 26/2 = -13. Absolute value = 13.

(5, 8)
(2, 6)
(-2, 12)
(5, 8)

Pass: 5C + X + 10, 5(13) + (168) + 10 = 243. **C = 13, Pass 243**

Seat D Pink Relay

Perform the long division and you get $2x^2 - 8x + 1$. The requested product is (2)(-8)(1) = -16.

Pass: X + 15D, 243 + 15(-16) = 3. **D = -16, Pass 3**

Seat E Pink Relay

$f(x) = \frac{x+3}{x-2}$. $f(f(x)) = \frac{\frac{x+3}{x-2}+3}{\frac{x+3}{x-2}-2} = \frac{\frac{x+3+3(x-2)}{x-2}}{\frac{x+3-2(x-2)}{x-2}} = \frac{4x-3}{-x+7}$. $f(f(f(x))) = \frac{\frac{4x-3}{-x+7}+3}{\frac{4x-3}{-x+7}-2} = \frac{\frac{4x-3+3(-x+7)}{-x+7}}{\frac{4x-3-2(-x+7)}{-x+7}} = \frac{x+18}{6x-17}$.
$\frac{x+18}{6x-17} = \frac{ax+b}{cx+d}$, a = 1, b = 18, c= 6, d = -17. ab – cd = 18 + 102 = 120.
Pass: EX = 120(3) = 360. **E = 120, Pass 360**

Seat A Yellow Relay

$8^k = 1/16$ ➔ $2^{3k} = 2^{-4}$ ➔ 3k = -4, k = -4/3. Pass: 24A, 24(-4/3) = -32. **A = -4/3, Pass -32**

Seat B Yellow Relay

$\frac{60+640+765+810+570+200}{35} = \frac{3045}{35} = 87$. Pass: B – 3X = 87 – 3(-32) = 183. **B = 87, Pass 183**

Seat C Yellow Relay

A(4, 3), B(8, 13), C(9, 1). Slope of $\overline{AB} = \frac{3-13}{4-8} = \frac{-10}{-4} = \frac{5}{2}$. Slope of $\overline{AC} = \frac{3-1}{4-9} = \frac{2}{-5}$. The slopes are negative reciprocals, so $\overline{AB} \perp \overline{AC}$, and A is the vertex of the right angle. AB = $\sqrt{10^2 + 4^2} = \sqrt{116} = 2\sqrt{29}$. AC = $\sqrt{(-5)^2 + 2^2} = \sqrt{29}$. Area = $\frac{1}{2}\left(2\sqrt{29}\cdot\sqrt{29}\right) = 29$.
Pass: $7(\frac{X}{3} - 2C)$, $7\left(\frac{183}{3} - 2(29)\right) = 7(61 - 58) = 21$. **C = 29, Pass 21**

Seat D Yellow Relay

By long division the quotient will be $3x^2 - 7x + 1$. The requested product is (3)(-7)(1) = -21.
Pass: X – 3D, 21 – 3(- 21) = 84. **D = -21, Pass 84**

Seat E Yellow Relay

f(x) = $\frac{x-2}{x+1}$. f(f(x)) = $\frac{\frac{x-2}{x+1}-2}{\frac{x-2}{x+1}+1} = \frac{\frac{x-2-2(x+1)}{x+1}}{\frac{x-2+x+1}{x+1}} = \frac{-x-4}{2x-1}$. f(f(f(x))) = $\frac{\frac{-x-4}{2x-1}-2}{\frac{-x-4}{2x-1}+1} = \frac{\frac{-x-4-2(2x-1)}{2x-1}}{\frac{-x-4+2x-1}{2x-1}} =$ $\frac{-5x-2}{x-5} = \frac{ax+b}{cx+d}$, a = -5, b = -2, c = 1, d = -5. $abcd$ = (-5)(-2)(1)(-5) = -50.

Pass: X – 8E, 84 – 8(-50) = 484. **E = -50, Pass 484**

Answer Sheet States 2025

ALT = Alternate Acceptable Answers

INDIVIDUAL

Individual Round 1	Individual Round 2	Individual Round 3
1. 2025 2. 10 3. 1200	1. 24 2. 8 3. 775	1. -27/5 ALT: $-5\frac{2}{5}$ *or* -5.4 2. 28:20 3. 65
Individual Round 4	**Individual Round 5**	**Individual Round 6**
1. 49 2. 72 3. (48, 42)	1. 66 2. ½ ALT: 0.5 *or* .5 3. 19	1. 294 2. $2\pi + 9$ ALT: $9 + 2\pi$ 3. 6

TEAM

Team Round 1		Team Round 2	
1. 49 2. 529 3. 11 4. $6\sqrt{3} + 4$ ALT: $4 + 6\sqrt{3}$	5. 48 6. $5\sqrt{5}$ 7. m^2/n^2 ALT: $\left(\frac{m}{n}\right)^2$ 8. 364	1. 50 2. 17 3. 127 4. -25	5. 236 6. 163 7. CHICKADEE 8. 36

RELAYS

BLUE		GREEN		PINK		GOLDENROD	
A = 12	Pass: 48	A = 11	Pass: 44	A = -2/3	Pass: -8	A = -4/3	Pass: -32
B = 11	Pass: 4	B = 35	Pass: 9	B = 88	Pass: 168	B = 87	Pass: 183
C = 90	Pass: 45	C = 65	Pass: 38	C = 13	Pass: 243	C = 29	Pass: 21
D = -36	Pass: 2	D = -35	Pass: - 73	D = -16	Pass: 3	D = -21	Pass: 84
E = 96	Pass: 24	E = 576	Pass: 65	E = 120	Pass: 360	E = -50	Pass: 484