Individuals Round 1 States 2024

3 pts 1. If $\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 7 \\ -1 & 8 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, find AB – CD. **Ans.** ___________

4 pts 2. Find the number of ordered pairs of positive integers (x, y) that satisfy the following equation: $x^y = 2^{64}$

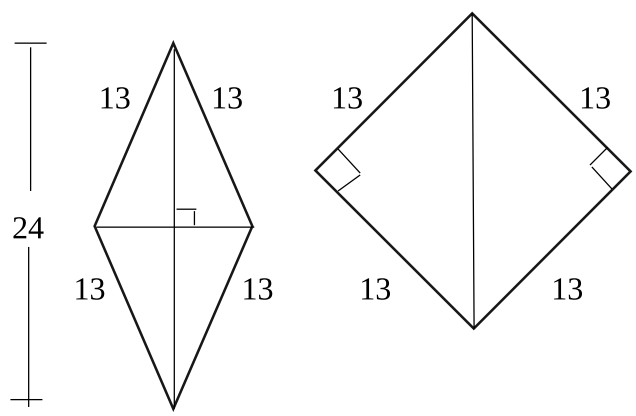
Ans. __________

5 pts 3. The zeroes of the equation of a parabola in the form $y = ax^2 + bx + c$, where a, b, and c are integers with no common factors and a > 0, are – 5/3 and 7/2. Find a + b + c.

Ans. _______________________________________

Individuals Round 2 States 2024

3 pts 1. What is the sum of the areas of the figures shown?



Ans. ___________________________

4 pts 2. Simplify $(a + b)^3 – (a – b)^3$, leaving no parentheses in the answer.

Ans. ________________________________

5 pts 3. If $f(x) = 3x$ and $g(x) = \frac{1}{x} – 3$, find the value of $[f^{-1}(g^{-1}(-1))]^{-1}$.

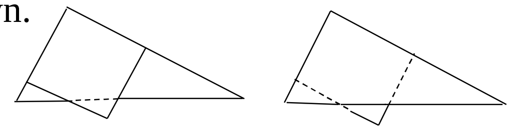
Ans. _____________________

Individuals Round 3 States 2024

3 pts 1. Let A represent the number of combinations of three different integers selected from the set {12, 13, 14, 15, 16, 17, 18, 19, 20}, such that the sum of the integers is even. Find A.

Ans. ______________________

4 pts 2. A right triangle and a square are arranged as shown. The square hides 80% of the triangle's area. When the layers are reversed, as shown in the right figure, only 15% of the square's area is visible. If the square measures 4 cm x 4 cm, what is the triangle's area?



Ans. ______________________

5 pts 3. Solve for x, if $\log_2 [\log_3 (\log_4 (x^{3x}))] = 0$.

Ans. ______________________

Individuals Round 4 States 2024

3 pts 1. The game of Yahtzee is played with five standard six-sided dice. A Yahtzee occurs when all five dice are rolled and all dice show the same number. What is the probability of rolling a Yahtzee on the first roll of all five dice?

Ans. ______________________

4 pts 2. If $\sqrt{144} + \sqrt[3]{216}$ can be written either as $\sqrt{a}$ or $6\sqrt{b}$, find the value of $\frac{a}{4b}$.

Ans. ______________________

5 pts 3. Peter has only pennies, Norma only nickels, Diane only dimes and Quincy only quarters. Peter and Norma have the same number of coins, and Diane and Quincy have the same number of coins. If T = the total number of coins they all have, what is the smallest value of T, such that, the sum of their money is \$4.87?

Ans. ______________________

Individuals Round 5 States 2024

3 pts 1. Let a two-digit number be called A. Reverse the digits to form a second two-digit number B. Find the largest value for A such that A:B = 4:7.

Ans. ___________________________

4 pts 2. Triangle ABC has vertices A (0, -5), B (-7, -6) and C (-3, -2). ΔA"B"C" is the image of ΔABC as the result of two transformations: (1) reflection over the y-axis, followed by (2) a 90° (counterclockwise) rotation about the origin. Let M = the sum of all the x-coordinates, and N = the sum of all the y-coordinates of the vertices of ΔA"B"C". Find the value of M + N.

Ans. ___________________________

5 pts 3. Find the sum of the solutions to the following equation:

$$|x - 14| = 0.25|x - 21| + 2.$$

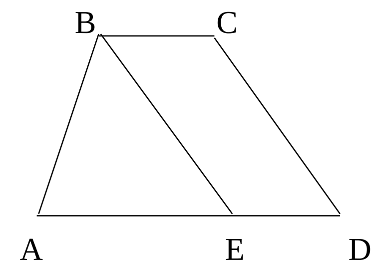
Ans. ___________________________

Individuals Round 6 States 2024

3 pts 1. We are given the following sequence:
PROBLEMSOLVINGPROBLEMSOLVINGPROB … . If the pattern continues, what letter will be in the 2024th position?

Ans. ______________________

4 pts 2. Trapezoid ABCD has side BC parallel to side AD as shown. $\overline{BE}$, which is parallel to $\overline{CD}$, creates a parallelogram and a triangle of equal areas. If AD = 10, find the length of $\overline{BC}$.



Ans. ______________________

5 pts 3. Find the area of the region bounded by the positive x-axis, the positive y-axis and the graph of y = -3$|x - 1|$ + 9.

Ans. ___________________________

Team Round 1 States 2024

1. a, b, c, d and e represent the first five terms, in order, of a sequence. Each term beginning with c is the sum of the previous two terms. If a = e = 9, find the value of d.

(1) Ans. ___________________4 pts

2. In a game of tug-of-war, 3 boys pull with the same force as 4 girls. One adult pulls with the same force as 2 girls and 1 boy. If 1 adult and 2 girls are tugging against n boys, how many boys are needed to balance the tug-of-war with no winner?

(2) Ans. ___________________4 pts

3. If x + y = 4, what is the value of the perimeter of a square with side length $2x^2 + 4xy + 2y^2$?

(3) Ans. ___________________6 pts

4. Art and Bob cross a lake with a single-seat kayak. Each can paddle at 7 mph and swim at 2 mph. They start together – one paddling, one swimming. Later, the kayaker anchors the kayak and starts swimming. When the swimmer reaches the kayak, he gets in and starts paddling. If the two arrive at their destination at the same time, what fractional part of the journey was the kayak anchored?

(4) Ans. ___________________6 pts

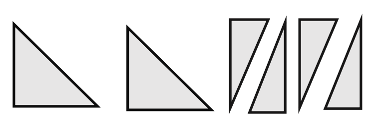
5. A positive integer is divided by 2, 3, 4 and 5, and the remainders are added together. Find the sum of the digits of the smallest integer that makes the sum as large as possible.

(5) Ans. ___________________6 pts

6. The numbers 220_b, 251_b and 304_b represent three consecutive perfect squares in base b. Determine the value of b.

(6) Ans. ______________8 pts

7. These six pieces of puzzle are put together in such a way to form a trapezoid. What is the trapezoid's perimeter?



On the left are two isosceles rt. Δ′s with legs of 13. On the right are four 5-12-13 rt. Δ′s. The definition of trapezoid that is used for this problem is "only two sides are parallel".

(7) Ans. ___________________8 pts

8. Two circles with radii of 25 and 16 are tangent externally. The two common external tangents intersect the larger circle at A and B and the smaller circle at C and D. The common internal tangent intersects the external tangents at E and F. Find the length of EF.

(8) Ans. ___________________8 pts

Team Round 2 States 2024

1. In how many ways can we select two different integers from the set {12, 13, … , 20} such that their sum is even?

(1) Ans. ______________4 pts

2. For five consecutive integers, the sum of the even integers is 60. What is the value of the smallest odd integer.

(2) Ans. ______________4 pts

3. A right triangle in the coordinate plane has vertices at A(1, 4), B(5, -1) and C(1, -1). The triangle is rotated 90^{o} counterclockwise about the origin, reflected over the y-axis, and then translated 4 units right and 3 units down. What are the coordinates of the midpoint of the hypotenuse of the triangle after the transformations?

(3) Ans. ______________6 pts

4. The dimensions of a right rectangular prism are in an arithmetic progression, with lateral edge greater than either base edge. The sum of the areas of the two bases is 5 ft^2, and the prism's volume is 10 ft^3. Find the lateral area of the prism.

(4) Ans. ___________________6 pts

5. For how many integers will the square of the integer decreased by twice the integer produce a difference of less than 4?

(5) Ans. ______________6 pts

6. How many positive integers between 2012_7 and 3012_7 are divisible by 20_7?

(6) Ans. ______________8 pts

7. The equation of a circle in the form $(x – h)^2 + (y – k)^2 = r^2$ contains the points (-5, 12), (9, 14) and (3, -4). Find the value of h + k + r.

(7) Ans. ______________8 pts

8. $\sin(\pi/7) + \sin(4\pi/7) + \sin(7\pi/7) + \sin(10\pi/7) + \sin(13\pi/7) + \sin(16\pi/7) =$ $\sin A$. Find the least value of A, where $0 \le A < 2\pi$ radians.

(8) Ans. ___________________8 pts

Seat A Blue Relay States 2024

The sum of two positive integers is 4 times the smaller integer. The positive difference of their squares is 8 times the larger integer. What is the product of the two intgers?

Pass back: A/3 A = Your answer

Seat B Blue Relay States 2024

The smallest integer greater than 1 that is a perfect square, a perfect cube, a perfect fourth power, a perfect fifth power, and a perfect sixth power is 2^n. Find n.

Pass back: B – X B = Your answer X = The number you will receive

Seat C Blue Relay States 2024

Point P lies on the perpendicular bisector of segment ST. If S has coordinates (5, 14), T has coordinates (-3, 2), and P has coordinates $(-a/2, a)$, what is the value of a?

Pass back: (X + 1)/C C = Your answer X = The number you will receive

Seat D Blue Relay States 2024

Let $k(p_n)$ = the smallest prime number greater than p_n. If p_1, p_2, and p_3 are the distinct prime factors of 2013, compute the following: $k(p_1) + k(p_2) + k(p_3) – k(p_1 + p_2 + p_3)$.

Pass back: DX D = Your answer X = The number you will receive

Seat E Blue Relay States 2024

Points Q (10,8) and R (24, 10) are the endpoints of the diameter of the circle centered at P. Line *m* is tangent to the circle at S (12, 14). What is the y-intercept of line *m*.

Pass in: X/E E = Your answer X = The number you will receive

Seat A Green Relay States 2024

The sum of two positive integers is 3 times the smaller integer. The positive difference of their squares is 6 times the larger integer. What is the product of the two integers?

Pass back: A/8 A = Your answer

Seat B Green Relay States 2024

The smallest integer greater than 1 that is a perfect square, a perfect cube, a perfect fourth power and a perfect sixth power is 2^n. Find n.

Pass back: B – X B = Your answer X = The number you will receive

Seat C Green Relay States 2024

Point P lies on the perpendicular bisector of segment ST. If S has coordinates (5, 14), T has coordinates (-3, 2), and P has coordinates (a, $3a/2$), what is the value of a?

Pass back: (C + 4)/X C = Your answer X = The number you will receive

Seat D Green Relay States 2024

Let $k(p_n)$ = the smallest prime number greater than p_n. If p_1, p_2, and p_3 are the distinct prime factors of 2037, compute the following: $k(p_1) + k(p_2) + k(p_3) – k(p_1 + p_2 + p_3)$.

Pass back: DX D = Your answer X = The number you will receive

Seat E Green Relay States 2024

Points Q (6, 8) and R (20, 10) are the endpoints of the diameter of a circle centered at P. Line m is tangent to the circle at S (8, 14). What is the y-intercept of line m?

Pass in: E/(X + 2) E = Your answer X = The number you will receive

Seat A Pink Relay States 2024

If 1/5 and 7/15 are the first and fifth terms of an arithmetic sequence, what is the sum of the second, third and fourth terms of the sequence?

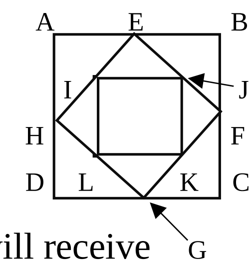
Pass back: 2A A = Your answer

Seat B Pink Relay States 2024

Nick has two part-time jobs that pay him by the hour. When he works twice as many hours at his day job as at his night job, his weekly pay is 4/5 of what he would earn if he reversed the number of hours worked at the two jobs. How many hours must Nick work at his day job to earn what he would earn in 10 hours at his night job?

Pass back: B/X B = Your answer X = The number you will receive

Seat C Pink Relay States 2024



ABCD is a square, and E, F, G, H, I, J, K, and L are midpoints of their respective line segments. Determine the area of IJKL, if the area of ABCD is 12 square units.

Pass back: X – C C = Your answer X = The number you will receive

Seat D Pink Relay States 2024

Arithmetic sequence $a_1, a_2, a_3, \dots, a_n$ has a common difference of 9. If the first and last terms of the sequence are – 58 and 77 respectively, find the following sum: $a_1 + a_2 + a_3 + \dots + a_n$.

Pass back: $\frac{D}{X+1}$ D = Your answer X = The number you will receive

Seat E Pink Relay States 2024

A triangle is inscribed in a circle. The ratio of the measures of the triangle's angles is 5:12 :13.

The shortest side of the triangle is a side of a regular n-gon inscribed in the circle. Determine n.

Pass in: (X - 1)/E E = Your answer X = The number you will receive

Seat A Yellow Relay States 2024

If 4/5 and 8/15 are the first and fifth terms of an arithmetic sequence, what is the sum of the second, third and fourth terms of the sequence.

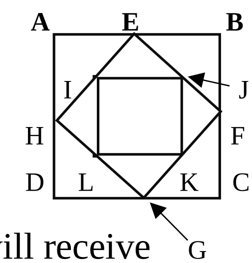
Pass back:2A A = Your answer

Seat B Yellow Relay States 2024

Nick has two part-time jobs that pay him by the hour. When he works twice as many hours at his day job as at his night job, his weekly pay is 4/5 what he would earn if he reversed the number of hours worked at the two jobs. How many hours must Nick work at his day job to earn what he would earn in 12 hours at his night job?

Pass back: B/X B = Your answer X = The number you will receive

Seat C Yellow Relay States 2024



ABCD is a square, and E, F, G, H, I, J, K, and L are midpoints of their respective line segments. Determine the area of IJKL, if the area of ABCD is 20 square units.

Pass back: X + C C = Your answer X = The number you will receive

Seat D Yellow Relay States 2024

Arithmetic sequence $a_1, a_2, a_3, \dots, a_n$ has a common difference of 8. If the first and last terms of the sequence are – 56 and 32 respectively, find the following sum: $a_1 + a_2 + a_3 + \dots + a_n$.

Pass back: D/(X + 1) D = Your answer X = The number you will receive

Seat E Yellow Relay States 2024

A triangle is inscribed in a circle. The ratio of the measures of the triangle's angles is 5 : 7 : 8.

The shortest side of the triangle is a side of a regular n-gon inscribed in the circle. Determine n.

Pass in: (X + 4)/E E = Your answer X = The number you will receive

Solutions – Individuals Round 1

1. $\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & -2\end{bmatrix}\cdot\begin{bmatrix}5 & 7\\ -1 & 8\\ 3 & -2\end{bmatrix} = \begin{bmatrix}20-2-3 & 28+16+2\\ 15-5-6 & 21+40+4\end{bmatrix} = \begin{bmatrix}15 & 46\\ 4 & 65\end{bmatrix}$. AB – CD = 15(46) – 4(65) = 690 – 260 = 430. **Ans. 430**

2. $x^y = 2^{64} = (2^2)^{32} = (2^4)^{16} = (2^8)^8 = (2^{16})^4 = (2^{32})^2 = (2^{64})^1$. So there are 7. **Ans. 7**

3. Using the zeroes x = -5/3 and x = 7/2 produces (3x + 5)(2x – 7) = 0 ➔ $6x^2 - 11x - 35 = 0$. So the sum a + b + c = 6 – 11 – 35 = -40. **Ans. – 40**

Individuals Round 2

1. The parallelogram's area = ½ $d_1 \cdot d_2$ = ½ (24)(10) = 120. The square's area is 169. **Ans. 289**

2. $(a + b)^3 - (a - b)^3$ ➔ $a^3 + 3a^2b + 3ab^2 + b^3 - (a^3 - 3a^2b + 3ab^2 - b^3)$ = **Ans. $6a^2b + 2b^3$**

3. f(x) = 3x, so $f^{-1}(x) = x/3$. g(x) = 1/x – 3, so $x = \frac{1}{g^{-1}(x)} - 3$ ➔ $g^{-1}(x) = \frac{1}{x+3}$. Thus

$[f^{-1}(g^{-1}(-1))]^{-1}$, $f^{-1}(\frac{1}{2})$ = 1/6 ➔ $[1/6]^{-1}$ = 6. **Ans. 6**

Individuals Round 3

1. The different combinations of three integers whose sum is even from {12, 13, 14, 15, 16, 17, 18, 19, 20} would have to be three even integers ${}_5C_3$ = 10 or two odd and one even, there are ${}_4C_2$ = 6 combinations of odd integers for five even integers: 5·6 = 30 ways. **Ans. 40**

2. 15% of 16 (the area of the square) is 2.4. 16 – 2.4 = 13.6 = 80% of the area of the triangle (which we will call A). A = 13.6/.8 = 17. **Ans. 17**

3. $\log_2 [\log_3 (\log_4 (x^{3x}))] = 0$ ➔ $\log_3 (\log_4(x^{3x}) = 1$ ➔ $\log_4(x^{3x}) = 3$ ➔ $x^{3x} = 64$, x = 2. **Ans.2**

Individuals Round 4

1. $(1/6)^5(6) = (1/6)^4 = \frac{1}{36\cdot 36}$ = 1/1296. **Ans. 1/1296**

2. $\sqrt{144} + \sqrt[3]{216}$ = 12 + 6 = 18. 18 = $\sqrt{324}$ or $6\sqrt{9}$. $\frac{a}{4b} = \frac{324}{36}$ = 9. **Ans. 9**

3. 6pn + 35dq = 487. The number of coins must be minimized, so dq must be maximized. If dq is 10, then 6pn = 137, which is not divisible by 6. If dq = 11, 6pn = 102, which is divisible by 6. The total number of coins is 22 + 34 = 56. **Ans. 56**

Individuals Round 5

1. A = 10t + u, B = 10u + t. $\frac{10t + u}{10u + t)} = \frac{4}{7}$ ➔ 70t + 7u = 40u + 4t ➔ 66t = 33u or 2t = u.
If u is 8, then t = 4. u cannot be 9 or 10. The largest value for the number is **Ans. 48**

2. The reflection of A (0, -5), B (-7, -6) and C (-3, -2) over the y-axis is A'(0, -5), B'(-7, -6), C'(3, -2). Rotation counter clockwise 90º: A"(5, 0), B"(6, 7), C"(2, 3). The sum is 23.

Ans. 23

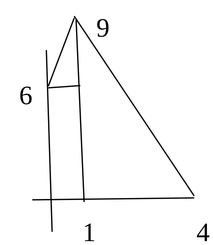
3. Consider the three cases: $x \leq 14$, $14 < x < 21$, and $x \geq 21$. In the third case, the equation becomes $x – 14 = .25(x – 21) + 2$. Solving for x produces a value of 43/3, contradicting the hypothesis that $x \geq 21$. If $x \leq 14$, the equation becomes $14 – x = 0.25(21 – x) + 2$, producing the solution x = 9. If $14 < x < 21$, the equation becomes $x – 14 = 0.25(21 – x) + 2$, producing the x = 17. Final answer: 9 + 17 = 26. **Ans. 26**

Individuals Round 6

1. The sequence repeats every 14 letters. 2024/14 = 144 r 8. Counting 8 gives S. **Ans. S**

2. Let x = BC = DE, so AE = 10 – x. aΔ ABE = ½ (10 -x)h, where h = height of triangle. The area of the parallelogram is xh. Thus ½ (10 -x)h = xh ➔ 10 - x = 2x. x = 10/3. **Ans. 10/3**

3. Graphing the region starts at the y intercept of 6. It reaches its highest point at (1, 9) and connects from there to one of the x intercepts of 4, which is found from solving the equation $3|x-1| = 9$, when letting y = 0. $x – 1 = \pm 3$ ➔ x = 4 or -2. Area: $6 + 3/2 + ½ (9\cdot 3) = 21$.



Ans. 21

Team Round 1

1. c = a + b = 9 + b. d = 9 + 2b. e = 18 + 3b = 9. So b = -3. d = 9 – 6 = 3. **Ans. 3**

2. Let b = boys, g = girls, a = adults. 3b = 4g and (1): a = 2g + b ➔ 2a = 4g + 2b ➔ 2a = 5b. 1a + 2g = (2g + b) + 2g = 4g + b = 3b + b = 4b. **Ans. 4**

3. $x + y = 4$. $2x^2 + 4xy + 2y^2 = 2(x + y)^2 = 32$. Perimeter = 4(32) = 128. **Ans. 128**

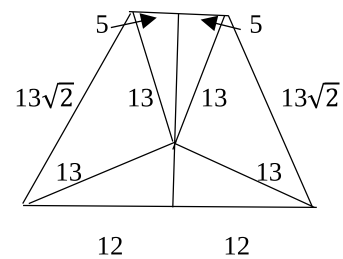
4. If one paddles for an hour, he goes 7 mi. The other has swims 2 mi. He needs to swim another 5 miles to get to the kayak, which will take him $2\frac{1}{2}$ hrs which is how long the kayak is anchored. He then needs to paddle another hour to get to the shore and the swimmer swims $3\frac{1}{2}$ hours to get there at the same time. So 2 ½ / 4 ½ = (5/2)/(9/2) = 5/9. **Ans. 5/9**

5. The least common multiple of 2, 3, 4, and 5 is 60. Subtracting 1 makes 59. The remainders are 1, 2, 3, 4 for 59, whose sum is 10. This may be the smallest number. Trying 49, 29 and19, we get 1, 1, 1,4 for 49; 1,2,1,4 for 29; 1,1,3,4 for 19. Other multiples 1 less than 5 are 54, 44, 34, 24, 14, none of these would maximize a 10, since they are all divisible by 2. **Ans. 14**

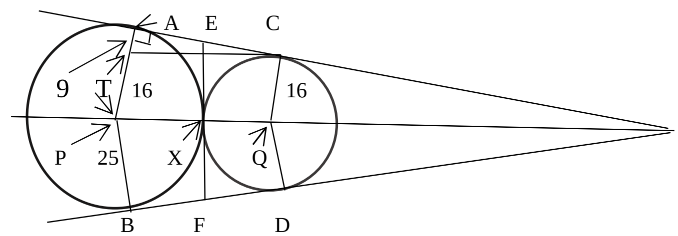
6. The base must be 6 or greater, since there is a 5 in one of the digits. $220_6 = 72 + 12 = 84$. $220_7 = 98 + 14 = 112$. $220_8 = 132 + 16 = 144$, a perfect square. $251_8 = 128 + 40 + 1 = 169$, another perfect square. $304_8 = 192 + 4 = 196$, also a perfect square. **Ans. 8**

7. The trapezoid at right is how it is put together.

The non-parallel sides are each $13\sqrt{2}$. The parallel sides are 10 on top and 24 on the bottom. $26\sqrt{2} + 34$. **Ans. $34 + 26\sqrt{2}$**



8. Refer to figure. AP = 25. CQ = 16. Draw CT parallel to PQ. PQ = 41 = TC, AT = 9 and AC = 40. AE = EX = EC = 20, since tangents to a circle from the same point are congruent. EX = 20, thus EF = 40. **Ans. 40**



Team Round 2

1. The number of 2 different integers from {12, 13, 14, 15, 16, 17, 18, 19, 20} which are even is ${}_5C_2 = 10$, if both are even. If both are odd we get ${}_4C_2 = 6$. Total = 16. **Ans. 16**

2. Let x, x + 1, x + 2, x + 3 and x + 4 be the integers. If x is odd, then x + 1 + x + 3 = 60. 2x = 56, so x = 28 and x + 1 = 29, contradictory. Thus x + x + 2 + x + 4 = 60, 3x = 54, x = 18. The smallest odd integer is 19. **Ans. 19**

3. The midpoint of segment AB, the hypotenuse is $((1 + 5)/2, (4 – 1)/2) = (3, 1\frac{1}{2})$. Rotating 90^o counterclockwise makes $(-1\frac{1}{2}, 3)$. Reflecting over the y-axis makes $(1\frac{1}{2}, 3)$. Translating 4 units right and 3 units down makes $(5\frac{1}{2}, 0)$. **Ans. $(5\frac{1}{2}, 0)$**

4. The bases are a and a + d and the height is a + 2d. (1) $a^2 + ad = 2.5$. The volume is $2.5(a + 2d) = 10$ ➔ (2) a + 2d = 4, this is the height. From this we get d = (4 – a)/2. Plugging this into (1) we get $a^2 + a\left(\frac{4-a}{2}\right) = 2.5$ ➔ $2a^2 + 4a – a^2 = 5$ ➔ $a^2 + 4a – 5 = 0$ ➔ (a + 5)(a – 1) = 0. So a = 1, since a cannot = -5. In (2) 1 + 2d = 4, so d = 3/2. The lateral area is 2(a + a+ d)(a + 4d) = $2(3\frac{1}{2})(4) = 8(\frac{7}{2}) = 28$. **Ans. 28**

5. Let m be the integer. Then we get $m^2 – 2m < 4$. Completing the square, we get $m^2 – 2m + 1 < 5$ or $(m – 1)^2 < 5$. Using m = -1, 0, 1, 2, 3, all will work. There are 5. **Ans. 5**

6. $2012_7 = 2 + 7 + 686 = 695$ base 10. $3012_7 = 2 + 7 + 1029 = 1038$ base 10. $20_7 = 14$ base 10. $695/14 = 49\frac{9}{14}$, so 50 is the first. $1038/14 = 74\frac{1}{7}$, 74 is the last. 74 – 50 + 1 = 25. **Ans. 25**

7. Using $x^2 + y^2 + ax + by + c = 0$: $25 + 144 - 5a + 12b + c = 0$ or (1) $-5a + 12b + c = -169$; $81 + 196 + 9a + 14b + c = 0$ or (2) $9a + 14b + c = -277$; $9 + 16 + 3a - 4b + c = 0$ or (3) $3a - 4b + c = -25$. (2) – (1): $14a + 2b = -108$ or (4) $7a + b = -54$. (2) – (3): $6a + 18b = -252$

(5): $a + 3b = -42 \Rightarrow -3(4) + (5) = -20a = 120$, so $a = -6$. In (5): $-6 + 3b = -42$, so $b = -12$. In (3): $3(-6) - 4(-12) + c = -25$, so $c = -55$. Therefore we get $x^2 + y^2 - 6x - 12y - 55 = 0 \Rightarrow$ $x^2 - 6x + 9 + y^2 - 12y + 36 = 45 + 55 = 100$ or $(x - 3)^2 + (x - 6)^2 = 10^2$. $h + k + r = 3 + 6 + 10 = 19$. **Ans. 19**

8. $\sin(\pi/7) + \sin(4\pi/7) + \sin(7\pi/7) + \sin(10\pi/7) + \sin(13\pi/7) + \sin(16\pi/7)$.

$\sin(10\pi/7) = \sin(\pi + 3\pi/7) = \sin\pi\cos 3\pi/7 + \cos\pi\sin 3\pi/7 = -\sin 3\pi/7$.
$\sin(13\pi/7) = \sin(\pi + 6\pi/7) = \sin\pi\cos 6\pi/7 + \cos\pi\sin 6\pi/7 = -\sin 6\pi/7$.
$\sin(\pi/7) = \sin(\pi - 6\pi/7) = \sin\pi\cos 6\pi/7 - \cos\pi\sin 6\pi/7 = \sin 6\pi/7$.
$\sin 4\pi/7 = \sin(\pi - 3\pi/7) = \sin\pi\cos 3\pi/7 - \cos\pi\sin 3\pi/7 = \sin 3\pi/7$. The first 4 terms cancel. $\sin(16\pi/7) = \sin(2\pi + \frac{2\pi}{7}) = \sin 2\pi\cos\frac{2\pi}{7} + \cos 2\pi\sin\frac{2\pi}{7} = \sin\frac{2\pi}{7}$. $A = \frac{2\pi}{7}$ or $\frac{5\pi}{7}$.

The least value is $2\pi/7$. **Ans. $\frac{2\pi}{7}$**

Seat A Blue Relay

Let $a < b$, then for the integers $a + b = 4a$, so $b = 3a$. $b^2 - a^2 = 8b$, so $(3a)^2 - a^2 = 8(3a) \Rightarrow$ $8a^2 = 24a$. Thus $a = 0$ or 3. If $a = 0$, and $b = 3a$, then $b = 0$. But $a < b$, so a cannot be 0. If $a = 3$, then $b = 9$. Their product is 27. Pass back: $A/3 = 9$. **A = 27, Pass: 9**

Seat B Blue Relay

The least common multiple of 2, 3, 4, 5, 6 is 60, so 2^{60} is the smallest integer. $2^{60} = (2^2)^{30} = (2^3)^{10} = (2^4)^{15} = (2^5)^{12} = (2^6)^{10}$. So $n = 60$. Pass back: $B - X = 51$. **B = 60, Pass: 51**

Seat C Blue Relay

The midpoint of $\overline{ST}$, if S(5, 14) and T(-3, 2) is (1,8). The slope of $\overline{ST}$ is $\frac{14-2}{5+3} = \frac{3}{2}$. The perpendicular bisector has slope $-\frac{2}{3}$ and passes through (1, 8): $8 = -\frac{2}{3}(1) + b$, so $b = 8\frac{2}{3}$. Its equation is $y = -\frac{2}{3}x + 8\frac{2}{3}$. For P, $a = -\frac{2}{3}(-\frac{a}{2}) + 8\frac{2}{3} \Rightarrow 3a = a + 26$, so $a = 13$.
Pass back: $(X + 1)/C = (51 + 1)/13 = 4$. **C = 13, Pass: 4**

Seat D Blue Relay

$2013 = 3\cdot 11\cdot 61$. $k(3) = 5$, $k(11) = 13$ and $k(61) = 67$. $k(75) = 79$. $5 + 13 + 67 - 79 = 6$. Pass back: $DX = 6(4) = 24$. **D = 6, Pass: 24**

Seat E Blue Relay

Center of diameter $\overline{QR}$ is (17, 9) which we will call T. Slope of TS is $\frac{9-14}{17-12}$ = - 1. So the slope of the tangent at S is 1. Its equation is y = x + b. Plug in (12, 14): 14 = 12 + b. So b = 2.
Pass in: X/E = 24/2 = 12. **E = 2, Pass: 12**

Seat A Green Relay

Let a < b for integers a and b. a + b = 3a or b = 2a. $b^2 - a^2 = 6b$. Substituting: $(2a)^2 - a^2 = 6(2a)$ $3a^2 - 12a = 0$. So a = 0 or 4. A cannot be 0, so b = 8, ab = 32.
Pass back: A/8 = 32/8 = 4. **A = 32, Pass: 4**

Seat B Green Relay

The LCM of 2, 3, 4, 6 is 12. B = 12, Pass back: B – X = 12 – 4 = 8. **B = 12, Pass: 8**

Seat C Green Relay

The midpoint of $\overline{ST}$, if S(5, 14) and T(-3, 2) is (1,8). The slope of $\overline{ST}$ is $\frac{14-2}{5+3} = \frac{3}{2}$. The perpendicular bisector has slope $-\frac{2}{3}$ and passes through (1, 8): $8 = -\frac{2}{3}(1) + b$, so $b = 8\frac{2}{3}$. Its equation is $y = -\frac{2}{3}x + 8\frac{2}{3}$. For P, $\frac{3}{2}a = -\frac{2}{3}(a) + 8\frac{2}{3}$ ➔ 9a = -4a + 52, so a = 4 .
Pass back: (X - 4)/C = (8 - 4)/4 = 4. **C = 4, Pass: 1**

Seat D Green Relay

2037 = 3· 7 · 97. k(3) = 5, k(7) = 11 and k(97) = 101. 5 + 11 + 101 = 117. k(3 + 7 + 97 = 107) = 109. 5 + 11 + 101 – 109 = 8. Pass back: DX = 8(1) = 8. **D = 8, Pass: 8**

Seat E Green Relay

Center of diameter $\overline{QR}$ is (13, 9) which we is P. Slope of $\overline{PS} = \frac{14-9}{8-13}$ = -1. So the slope of the tangent at S is 1. Its equation is y = x + b. Plug in (8, 14): 14 = 8 + b. So b = 6.
Pass in: E/(X + 2) = 6/(8 + 2) = 3/5. **E = 6, Pass: 3/5**

Seat A Pink Relay

1/5 = 3/15, next three terms are 4/15, 5/15, 6/15. Their sum = 1. Pass back: 2A = 2.

A = 1, Pass: 2

Seat B Pink Relay

Let d = day-job hourly rate and n = night-job hourly rate. $2d + n = \frac{4}{5}(2n + d)$ ➔ 10d + 5n = 8n + 4d ➔ 6d = 3n or 2d = n. Nick gets paid twice as much at his night job as at his day job. So he must work 20 hours at his day job to make 10 hours of his night job. Pass back: B/X = 20/2 = 10. **B = 20, Pass: 10**

Seat C Pink Relay

AB = $\sqrt{12}$ = $2\sqrt{3}$, so AE = $\sqrt{3}$. HE = $\sqrt{6}$, so BI = $\sqrt{6}/2$. LI = $\sqrt{2}\left(\frac{\sqrt{6}}{2}\right)$ = $\sqrt{3}$, so the area of IJKL = 3. Pass back: X – C = 10 – 3 = 7. **C = 3, Pass: 7**

Seat D Pink Relay

$a_n = a_1 + (n-1)d$ ➔ 77 = -58 + (n – 1)9 ➔ 135 = 9n – 9, 16 = n. Sum = $16\left(\frac{-58+77)}{2}\right)$ = 8(19) = 152. Pass back: $\frac{D}{X+1} = \frac{152}{7+1}$ = 19. **D = 152, Pass: 19**

Seat E Pink Relay

The sum of the angles of a triangle 180º, so 5x + 12x + 13x = 180 ➔ 30x = 180, so x = 30. The arc opposite the inscribed angle is 60º. 360º/60º = 6 sides. Pass in: $\frac{X-1}{E}$ = 18/6 = 3.
E = 6, Pass: 3

Seat A Yellow Relay

4/5 = 12/15. The middle three terms are 9/15, 10/15, 11/15. The sum of these is 2.
Pass back: 2A = 2(2) = 4. **A = 2, Pass: 4**

Seat B Yellow Relay

Refer to Pink Relay Seat B. Nick would have to work 24 hours to make up for 12 night job hours. Pass back: B/X = 24/4 = 6. **B = 24, Pass: 6**

Seat C Yellow Relay

AB = $\sqrt{20}$, so AE = $\sqrt{5}$. AE = $\sqrt{10}$, so HI = $\frac{\sqrt{10}}{2}$. IL = $\frac{\sqrt{20}}{2}$ = $\sqrt{5}$. The area of IJKL = 5. Pass back: C + X = 5 + 6 = 11. **C = 5, Pass: 11**

Seat D Yellow Relay

$a_n = a_1 + (n-1)d$ ➔ 32 = - 56 + (n – 1)8 ➔ 88 = 8n – 8, 12 = n. Sum = $12\left(\frac{-56+32)}{2}\right)$ = 6(-24) = -144. Pass back: $\frac{D}{X+1} = \frac{-144}{11+1}$ = -12. **D = - 144, Pass: - 12**

Seat E Yellow Relay

As in Pink Relay Seat E, 5x + 7x + 8x = 180º ➔ 20x = 180, x = 9. So 5x = 45. So the arc opposite the smallest angle measures 90º, so the regular polygon is a square with 4 sides.

Pass in: (X + 4)/E = (-12 + 4)/4 = -2. **E = 4, Pass: - 2**

Answer Sheet States 2024

Individuals Round 1

1. 430
2. 7
3. – 40

Individuals Round 2

1. 289
2. $6a^2b + 2b^3$
3. 6

Individuals Round 3

1. 40
2. 17 ($17cm^2$)
3. 2

Individuals Round 4

1. 1/1296
2. 9
3. 56

Individuals Round 5

1. 48
2. 23
3. 26

Individuals Round 6

1. S
2. $\frac{10}{3}$ $(3\frac{1}{3}, 3.\overline{3})$
3. 21

Team Round 1

1. 3
2. 4
3. 128
4. 5/9
5. 14
6. 8
7. $34 + 26\sqrt{2}$
8. 40

Team Round 2

1. 16
2. 19
3. $(5\frac{1}{2}, 0)$
4. 28
5. 5
6. 25
7. 19
8. $\frac{2\pi}{7}$

or $[(5.5, 0), (\frac{11}{2}, 0)]$

Blue Relay	Green Relay
A = 27, Pass 9	A = 32, Pass 4
B = 60, Pass 51	B = 12, Pass 8
C = 13, Pass 4	C = 4, Pass 1
D = 6, Pass 24	D = 8, Pass 8
E =2, Pass 12	E = 6, Pass 3/5 (0.6)

Pink Relay	Yellow Relay
A = 1, Pass 2	A = 2, Pass 4
B = 20, Pass 10	B = 24, Pass 6
C = 3, Pass 7	C = 5, Pass 11
D = 152, Pass 19	D = - 144, Pass - 12
E = 6, Pass 3	E = 4, Pass – 2