Round 1 Individuals States 2016

3 pts 1. Of 100 people in a room, 70 play tennis and 50 play golf. Every person plays one or the other or both sports. How many play only one sport?

Ans. ____________________

4 pts 2. Find all real values of x for which $\left(\log x^2\right)^2 + \log x^2 = 2$.

Ans. ___________________________

5 pts 3. What is the largest prime factor of 8091?

Ans. ____________________

Round 2 Individuals States 2016

3 pts 1. If the numerical value of the area of a square plus two times the numerical value of its perimeter is equal to 20. What is the area of the square?

Ans. ____________________

4 pts 2. Find the square root of the sum of the first 87 positive odd integers.

Ans. _________________________

5 pts 3. Suppose the volume of a sphere is equal to $\frac{\pi}{6}$ times the volume of a cube. What is the ratio of the surface area of the sphere to the surface area of the cube?

Ans. _________________________

Round 3 Individuals States 2016

3 pts 1. Nora has $4.70 consisting of nickels, dimes and quarters. She has 3 times as many nickels as quarters, and 16 more nickels than dimes. What is the value of all of the dimes in dollars and cents?

Ans. ___________________

4 pts 2. What is the sum of the digits of the integer solution to the equation

$$\sqrt{14+\sqrt{27-\sqrt{x-1}}} = 4?$$

Ans. ________________________

5 pts 3. If x – 1, 2x + 3, and 5x – 1 are the first three terms of an arithmetic sequence, compute the numerical value of the 17^{th} term of this sequence.

Ans. _________________________

Round 4 Individuals States 2016

3 pts 1. A rectangular swimming pool is twice as long as it is wide and is surrounded by a 10 ft wide concrete border. If the concrete border has an area of 2800 square feet, what is the perimeter of the pool?

Ans. _________________________

4 pts 2. If log (x + 1) + log (x + 2) = log (2x + 22), solve for x.

Ans. _________________________

5 pts 3. Find all values of x where $0^o \le x < 360^o$ and sin x = cos 2x.

Ans. ______________________________

Round 5   Individuals   States 2016

3 pts 1. If the number 86 in base ten is represented as 321 in base *b*, what is the base ten representation of the base *b* number 123?

Ans. ______________________

4 pts 2. An "annulus" is a region bounded by two concentric circles. A chord of the larger circle is tangent to the smaller of two concentric circles. If the chord has a length of 12, what is the area of the "annulus" of the circles?

Ans. ______________________

5 pts 3. Simplify: $\sqrt{3+2\sqrt{2}}-\sqrt{3-2\sqrt{2}}$. Express your answer in simplest form.

Ans. ___________________________

Round 6   Individuals   States 2016

3 pts 1. In a bin at the Cumberland Convenience Store there are 200 candies. Of these candies, 90% are red and 10% are gold. After Bob buys some of the red candies, 80% of the remaining candies in the bin are red. How many candies did Bob buy?

Ans. __________________

4 pts 2. If p, q, and r are positive integers, and $p+\dfrac{1}{q+\dfrac{1}{r}}=\dfrac{25}{19}$, find q.

Ans. _________________

5 pts 3. Consider the circles that have radii $4\sqrt{5}$ and are tangent to the line x – 2y = 20 at the point (6, -7). Find the sum of the x coordinates of the centers of the circles.

Ans. ______________________

Round 1 Team States 2016

4 pts 1. A fair coin is flipped and a fair six-sided die is rolled. What is the probability of getting a tail and a prime number.

(1) Ans. ______________________4 pts

4 pts 2. If you add the denominator, 3, to both the denominator and numerator of the fraction 1/3, the value of the fraction will double. Find the fraction that will triple in value when its denominator is added to its numerator and its denominator.

(2) Ans. ______________________4 pts

6 pts 3. What is the sum of the coefficients and constant in the expansion of $(3-4x)^5$?

(3) Ans. ______________________ 6 pts

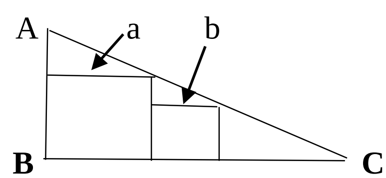
6 pts 4. A ladder leans against a house with its base 15 ft away from the house. When the base of the ladder is pulled 9ft further away from the house, the upper end of the ladder slides down 13 ft. How long is the ladder?

(4) Ans. ______________________ 6 pts

6 pts 5. For some fixed constant(s), *b*, the statement sin x = cos (x + *b*) is an identity that is true for all x. Find the sum of all possible values of *b* in radians for $0 \le b < 2\pi$.

(5) Ans. ______________________ 6 pts

8 pts 6. Given $30^o - 60^o - 90^o \Delta ABC$ with inscribed squares of side lengths *a* and *b*, m∠A = 60º, m∠C = 30º, find *b*/*a* in simplest radical form.



(6) Ans. ________________________8 pts

8 pts 7. Suppose the roots of the quadratic equation $x^2 + ax + b$ are sin 15^0 and cos 15^0. What is the value of $a^4 - b^2$?

(7) Ans. ___________________________ 8 pts

8 pts 8. Two circles whose centers are 10 cm apart have a common external tangent segment of length 8 cm and a common internal tangent segment of length $\sqrt{34}$ cm. Determine the product of the two radii.

(8) Ans. _______________________ 8 pts

Round 2   Team   States 2016

4 pts 1. Coming out of a grocery store, Ed had eight U.S. coins each of which is under 50 cents. All eight were worth \$1.45. Unfortunately on the way home he lost one of them. What is the probability that the coin he lost was a nickel?

(1) Ans. ___________________4 pts

4 pts 2. If the quadratic equation $x^2 + bx + c = 0$ has exactly one solution, *r*. Find b/c in terms of *r*.

(2) Ans. __________________ 4 pts

6 pts 3. Find all real numbers x such that $25|x| = x^2 + 144$?

(3) Ans. __________________ 6 pts

6 pts 4. If the length of the hour hand and minute hands of an analog clock are 4 cm and 6 cm respectively, what is the distance in cm between the tips of the hands at two o'clock? Give an exact answer in simplest form.

(4) Ans. __________________ 6 pts

6 pts 5. Frick and Frack want to paint a fence. Frick can paint the fence alone in 3 hrs. Frack can paint it alone in 4 hrs. At noon they start painting the fence together. At some point they fight for ten minutes and no painting gets done. After the fight Frack leaves and Frick paints what is left. If Frick finishes at 2:40 pm. At what time did the fight begin?

(5) Ans. __________________ 6 pts

8 pts 6. Determine the value of the infinite expression: $\sqrt{20+\sqrt{20+\sqrt{20+...}}}$.

(6) Ans. ________________________ 8 pts

8 pts 7. A square has its base on the x axis and one vertex on each branch of the curve y = $\frac{1}{x^2}$.

Find its area. Give your answer in simplest radical form.

(7) Ans. ______________________________ 8 pts

8 pts 8. Find the sum of the elements in the 100th row of this triangular array:

1

2 3

4 5 6

7 8 9 10

(7) **Ans. _______________________ 8 pts**

Blue Relay Seat A States 2016

If the graphs of 3x – 5y + 4 = 0 and 2x + ay – 11 = 0 meet at right angles, then what is the value of a?

Pass back: 15 A A = Your Answer.

Blue Relay Seat B States 2016 States 2016

The Jones's and Phelps families each started from a rest area on Interstate 84 heading south, the Jones's at 80 mph and the Phelps at 65 mph. The Phelps had to stop at another rest area for half an hour, and then continued at the same rate. How far have the Phelps traveled when they are 70 miles apart?

Pass back: 100X + B + 67 B = Your Answer X = TNYWR

Blue Relay Seat C States 2016 States 2016

A sphere of radius 25 cm is intersected by a plane passing 24 cm from its center. Find the area of the circle of intersection in terms of π.

Pass back: $X - \frac{C}{\pi}$ C = Your Answer X = TNYWR

Blue Relay Seat D States 2016 States 2016

The fourth term of a positive geometric series is 20. The sixth term is 6!. What is the fifth term?

Pass back: $\frac{X}{4} - D$ D = Your Answer X = TNYWR

Blue Relay Seat E States 2016 States 2016

How many base 6 numbers are 4 digits long?

Pass in: 3X – E E = Your Answer X = TNYWR

Green Relay Seat A States 2016 **States 2016**

If the graphs of the equations 5x – 3y – 4 = 0 and 2x + *a*y – 11 = 0 intersect at right angles, find the value of *a*.

Pass back: 15A A = Your Answer.

Green Relay Seat B States 2016 **States 2016**

The Jones and Phelps families each started from a rest area on Interstate 84 heading south, the Jones's at 80 mph and the Phelps at 65 mph. The Phelps had to stop at another rest area for half an hour, and then continued at the same rate. How far have the Jones's traveled when they are 70 miles apart?

Pass back: 8(X + B) – 44 B = Your Answer X = TNYWR

Green Relay Seat C States 2016

A sphere of radius 17 cm is intersected by a plane passing 15 cm from its center. Find the area of the circle of intersection in terms of π.

Pass back: $X - \frac{C}{\pi}$ C = Your Answer X = TNYWR

Green Relay Seat D States 2016

The fifth term of a geometric series is 5! And the sixth term is 6! What is the fourth term?

Pass back: $\frac{X}{4} - D$ D = Your Answer X = TNYWR

Green Relay Seat E States 2016

How many base 5 numbers are 5 digits long?

Pass in: E – 5X E = Your Answer X = TNYWR

Pink Relay Seat A States 2016

The mean of a and b is x. The mean of b and x equals the mean of a and $(b + 1)$. Find $a - b$.

Pass back: 8 – A A = Your Answer

Pink Relay Seat B States 2016

Alex is 10 years older than Ben. In 12 years Ben will be 2/3 as old as Alex. How old is Alex?

Pass back: B – X B = Your Answer X = TNYWR

Pink Relay Seat C States 2016

A rhombus has sides of length 15 and its diagonals differ by 6. What is its area?

Pass back: $\frac{C}{X}$ C = Your Answer X = TNYWR

Pink Relay Seat D States 2016

So that zip codes may be machine readable, the U.S. Postal Service encodes them as a group of five bars, some tall, some short. In how many distinct ways can two tall and three short bars be arranged?

Pass back: DX D = Your Answer X = TNYWR

Pink Relay Seat E States 2016

Solve the following: $\frac{2^{14x+11}}{8} \cdot \frac{4^{21x-1}}{32} = \frac{8^{7x+10}}{128}$

Pass in: 7EX E = Your Answer X = TNYWR

Yellow Relay Seat A States 2016

The mean of a and b is x. The mean of b and x equals the mean of a and $(b-2)$. Find $a-b$.

Pass back: 8 – A A = Your Answer

Yellow Relay Seat B States 2016

Alex is 10 years older than Ben. In 12 years Ben will be 2/3 as old as Alex. How old is Ben?

Pass back: B – X B = Your Answer X = TNYWR

Yellow Relay Seat C States 2016

A rhombus has sides of length 10 and its diagonals differ by 4. What is its area?

Pass back: $\frac{C}{X}$ C = Your Answer X = TNYWR

Yellow Relay Seat D States 2016

So that zip codes may be machine readable, the U.S. Postal Service encodes them as a group of five bars, some tall, some short. In how many distinct ways can one tall and four short bars be arranged?

Pass back: DX D = Your Answer X = TNYWR

Yellow Relay Seat E States 2016

Solve the following: $\frac{2^{14x+11}}{\sqrt{2}} \cdot \frac{4^{21x-1}}{32} = \frac{8^{7x+10}}{128}$

Pass in: (X + 20)E E = Your Answer X = TNYWR

Solutions – Round 1   Individuals

1. Let x be those that play both sports. Then (70 – x) + (50 – x) + x = 100. 120 – x = 100. So x = 20. Therefore those that don't play both is 80. **Ans. 80**

2. $\left(\log x^2\right)^2 + \log x^2 = 2$ ➔ $\left(\log x^2\right)^2 + \log x^2 - 2 = 0$ ➔ $(\log x^2 + 2)(\log x^2 - 1) = 0$.

If $\log x^2 + 2 = 0$, then $\log x^2 = -2$ ➔ $x^2 = 1/100$. So $x = \pm 1/10$. If $\log x^2 - 1$, then $x^2 = 10$. So $x = \pm\sqrt{10}$. **Ans.** $\pm\sqrt{10}$, $\pm 1/10$

3. 8091 = 9(899). 899 = 900 – 1 = (30 + 1)(30 – 1) = 31(30). LPF = 31. **Ans. 31**

Round 2   Individuals

1. $s^2 + 2(4s) = 20$ ➔ $s^2 + 8s - 20 = 0$ ➔ $(s + 10)(s - 2) = 0$. s = 2, so $s^2 = 4$. **Ans. 4**

2. First odd is 1, first 2 odd is 1 + 3 = 4, first three odd is 1 + 3 + 5. The pattern first 1 is 1, second is 4, third is 9. So the first 87 positive odd integers is 87^2. $\sqrt{87^2} = 87$. **Ans. 87**

3. $\frac{4}{3}\pi r^3 = \frac{\pi}{6}s^3$ ➔ $\frac{r^3}{s^3} = \frac{\pi}{6}\cdot\frac{3}{4\pi} = \frac{1}{8}$, so $\frac{r}{s} = \frac{1}{2}$. Ratio of surface areas: $\frac{4\pi r^2}{6(2r)^2} = \frac{4\pi r^2}{24r^2} = \frac{\pi}{6}$. **Ans.** $\frac{\pi}{6}$

Round 3   Individuals

1. 5(3x) + 10(3x – 16) + 25x = 470 ➔ 15x + 30x – 160 + 25x = 470 ➔ 70x = 630, so x = 9. There are 3(9) – 16 = 11 dimes. The value is $1.10. **Ans. $1.10**

2. $\sqrt{14+\sqrt{27-\sqrt{x-1}}} = 4$ ➔ $14 + \sqrt{27-\sqrt{x-1}} = 16$ ➔ $\sqrt{27-\sqrt{x-1}} = 2$ ➔ $27 - \sqrt{x-1} = 4$ ➔ $\sqrt{x-1} = 23$ ➔ x – 1 = 529, so x = 530. Sum of digits is 8. **Ans. 8**

3. (2x – 3) – (x – 1) = (5x – 1) – (2x – 3) ➔ x – 2 = 3x + 2, so x = 4. The first three terms are 3, 11, and 19. Common difference is 8. 17^{th} term is 3 + 16(8) = 3 + 128 = 131. **Ans. 131**

Round 4   Individuals

1. Let the width of the pool be w and the length be 2w, then (w + 20)(2w + 20) – 2(2w) = 2800. $2w^2 + 60w + 400 - 2w^2 = 2800$ ➔ 60w = 2400, so w = 40, l = 80. Perimeter = 240. **Ans. 240**

2. (x + 1)(x + 2) = 2x + 22 ➔ $x^2 + 3x + 2 - 2x - 22 = 0$ ➔ $x^2 + x - 20 = 0$. (x + 5)(x – 4) = 0. Thus x = 4. **Ans. 4**

3. sin x = cos 2x ➔ $\sin x = 1 - 2\sin^2 x$ ➔ $2\sin^2 x + \sin x - 1 = 0$ ➔ (2 sin x – 1)(sin x + 1) = 0 sin x = ½ at 30° and 150°. sin x = -1 at 270°. **Ans. 30°, 150°, 270°**

Round 5 Individuals

1. $3b^2 + 2b + 1 = 86$ ➔ $3b^2 + 2b - 85 = 0$ ➔ $(3b + 17)(b - 5) = 0$, so b = 5. 123 in base 5 makes 1(25) + 2(5) + 3(1) = 25 + 10 + 3 = 38. **Ans. 38**

2. Let r be the radius of the smaller circle and r + x the radius of the larger circle. Then $r^2 + 6^2 = (r+x)^2$ ➔ Area needed: $\pi(r+x)^2 - \pi r^2 = \pi(r^2 + 6^2) - \pi r^2 = 36\pi$. **Ans. 36$\pi$**

3. Let x = $\sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$, then $x^2 = 3 + 2\sqrt{2} - 2\sqrt{9-8} + 3 - 2\sqrt{2} = 6 - 2 = 4$. x = 2. **Ans. 2**

Round 6 Individuals

1. .9(200) = 180 reds, 180 – x = .8(200 – x) ➔ 180 – x = 160 – .8x ➔ 20 = .2x. **Ans. 100**

2. In $p + \dfrac{1}{q + \frac{1}{r}} = \dfrac{25}{19}$, p = 1 and thus $\dfrac{1}{q + \frac{1}{r}} = \dfrac{6}{19}$. $q + \dfrac{1}{r} = \dfrac{qr+1}{r} = \dfrac{19}{6}$, thus q = 3. **Ans. 3**

3. The line perpendicular to x – 2y = 20 at the point (6, -7) is 2x + y = 5. If we use the parallel lines to x – 2y = 20 through the two radii, we can find the x coordinates where they intersect the perpendicular line (1) 2x + y = 5. Using the distance between parallel lines, which is $\dfrac{|F|}{\sqrt{a^2+b^2}} = 4\sqrt{5} = \dfrac{|F|}{\sqrt{1^2+2^2}}$, $|F|$ = 20. This produces the two lines parallel to x – 2y = 20 which are (2) x – 2y = 0 and (3) x – 2y = 40. 2(1) + (2): 5x = 10, x = 2. 2(1) + (3): 5x = 50, x = 10.

Ans. 12

Round 1 Team

1. P(tail) = 1/2. P(Prime) = 1/2. (1/2)(1/2) = 1/4. **Ans. 1/4**

2. Let $\dfrac{n}{d}$ be the fraction. Then $\dfrac{3n}{d} = \dfrac{n+d}{2d}$ ➔ $\dfrac{6n}{2d} = \dfrac{n+d}{2d}$ ➔ 5n = d ➔ $\dfrac{5n}{d} = 1$ ➔ $\dfrac{n}{d} = \dfrac{1}{5}$. **Ans. 1/5**

3. $(3)^2 + 5(3)^4(-4) + 10(3)^3(-4)^2 + 10(3)^2(-4)^3 + 5(3)(-4)^4 + (-4)^5 =$

243 – 1620 + 4320 – 5760 + 3840 – 1024 = -1. **Ans. -1**

4. Let L = length of ladder, H = height up the building that ladder rests. At start $L^2 = H^2 + 225$.

After 13 foot drop: $L^2 = (H - 13)^2 + 576$. $H^2 - 26H + 169 + 576 = H^2 + 225$ ➔ 520 = 26H.

H = 20. $L^2 = 400 + 225 = 625$. L = 25. **Ans. 25**

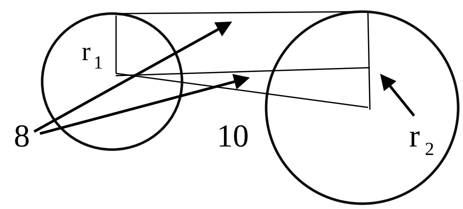
5. The phase shift for cos (x + b) is b. The value of b is the number of degrees y = cos x has to go through from 0° to 360° or in this case 0 to 2π radians before it starts to generate the graph of y = sin x. When it gets to 270° or $\dfrac{3\pi}{2}$ radians it starts. **Ans. $\dfrac{3\pi}{2}$**

6. The end of the long leg is $b\sqrt{3}$. The end of the short leg is $\frac{a}{\sqrt{3}}$. The long leg is $\sqrt{3}$ times the short leg: $a + b + b\sqrt{3} = \sqrt{3}\left(a+\frac{a}{\sqrt{3}}\right)$ ➔ $a + b + b\sqrt{3} = a\sqrt{3} + a$ ➔ $b(1 + \sqrt{3}) = a\sqrt{3}$.

Therefore $\frac{b}{a}=\frac{\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}=\frac{\sqrt{3}-3}{1-3}=\frac{3-\sqrt{3}}{2}$. **Ans.** $\frac{3-\sqrt{3}}{2}$

7. The sum of the roots is $-a = \sin 15^o + \cos 15^o$. $(-a)^2 = a^2 =$ $\sin^2 15^o + 2\sin 15^o \cos 15^o + \cos^2 15^o = 1 + \sin 30^o$ = 1 + 1/2 = 3/2. So $(3/2)^2 = 9/4 = a^4$. The product of the roots is $b = \sin 15^o \cos 15^o$, so $2b = 2\sin 15^o \cos 15^o = \sin 30^o = 1/2$, so b = 1/4, and $b^2 = 1/16$. $a^4 - b^2 = 9/4 - 1/16 = 35/36$. **Ans. 35/16**

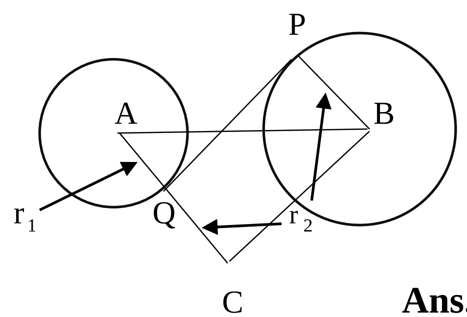
8. At right: $(r_2 - r_1)^2 + 64 = 100$ or (1) $r_2^2 - 2r_1r_2 + r_1^2 = 36$.



At right: connect B to P making right angle BPQ; PB = r_2.

Extend AQ a distance r_2 long and then connect B to C making right triangle ACB. PQ = BC = $\sqrt{34}$. Now

$(r_1 + r_2)^2 + 34 = 100$ ➔ (2) $r_1^2 + 2r_1r_2 + r_2^2 = 66$.



(2) – (1) = $4r_1r_2 = 30$, so $r_1r_2 = 7.5$ **Ans. 7.5**

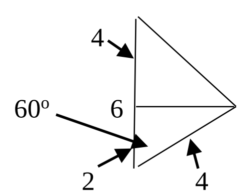
Round 2 Team

1. The 8 coins are 5 quarters, 1 dime and 2 nickels. P(a nickel) = 2/8. **Ans. 1/4**

2. If the solution is r, then x = r, or $(x - r)(x - r) = x^2 - 2rx + r^2 = 0$. $\frac{b}{c}=\frac{-2r}{r^2}=\frac{-2}{r}$. **Ans.** $\frac{-2}{r}$

3. $x^2 + 144 = \pm 25$ ➔ (1) $x^2 + 25x + 144 = 0$ or (2) $x^2 - 25x + 100 = 0$. In (1) (x-9)(x-16) = 0. So x = 9 or 16, in (2) x = -9 or -16. **Ans. $\pm$9, $\pm$16**

4. In the figure, drop a perpendicular from C to AB at D. In $\triangle$ADC, AD = 2, and DC = $2\sqrt{3}$. Since BD = 4, then $BC^2 = 16 + 12 = 28$.



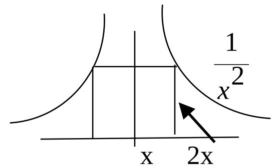
BC = $2\sqrt{7}$. **Ans. $2\sqrt{7}$**

5. Let x = hrs Frack paints. $\frac{2.5}{3}+\frac{x}{4}=1$ ➔ 10 + 3x = 12, x = 2/3 = 40 min. **Ans. 12:40 pm**

6. If x = $\sqrt{20+\sqrt{20+\sqrt{20+...}}}$, then x^2 = 20 + $\sqrt{20+\sqrt{20+\sqrt{20+...}}}$ ➔ x^2 = 20 + x.

x^2 – x – 20 = 0 ➔ (x – 5)(x + 4) = 0, so x = 5. **Ans. 5**

7. In the figure, if from the origin you go a distance x, then you have to go a distance of 2x on the y axis to make a square. Therefore



2x = $\frac{1}{x^2}$ ➔ x^3 = 1/2, so x = $\sqrt[3]{\frac{1}{2}}$. The area is $\left(2\cdot\sqrt[3]{\frac{1}{2}}\right)^2=4\cdot\sqrt[3]{\frac{1}{4}\cdot\frac{2}{2}}=2\sqrt[3]{2}$. **Ans.** $2\sqrt[3]{2}$

8. For each row, there are n numbers in each. The last number in each row can be found by using $\frac{n(n+1)}{2}$. So the 100th triangular number is $\frac{100(100+1)}{2}=5050$. The first number in the row is (5050 – 100 + 1) = 4951. The sum = $\frac{100}{2}(4951+5050)=50(10001)$. **Ans. 500,050**

Blue Relay Seat A

The line perpendicular to 3x – 5y = -4 has form 5x + 3y = c. So 5x + 3y = 2x + ay ➔ $\frac{5}{3}=\frac{2}{a}$

5a = 6, so a = 6/5. Pass back:15A = 15(6/5) = 18. **A = 6/5; Pass 18**

Blue Relay Seat B

Jones: 80T = D

Phelps: 65(T – 1/2) = D – 70

65(T – 1/2) = 80T – 70 ➔ 37½ = 15T, so T = 2½ . 65(2) = 130

Pass back: 100X + B + 67 = 100(18) + 130 + 67 = 1997.

B = 130; Pass 1997

Blue Relay Seat C

By Pythagorean Theorem $r^2 + 24^2 = 25^2$, so r = 7. Area = 49π .

Pass back: $X-\frac{C}{\pi}$ = 1997 - $\frac{49\pi}{\pi}$ = 1948. **C = 49π ; Pass 1048**

Blue Relay Seat D

6! = 720. $20r^2 = 720$, $r^2 = 36$, so r = 6. 20(6) = 120. Pass back:

$\frac{X}{4}-D=\frac{1948}{4}-120=487-120=367$. **D = 120; Pass 367**

Blue Relay Seat E

There should be $5 \cdot 6 \cdot 6 \cdot 6 = 1080$ numbers. Pass in: 3X –E: 3(367) – 1080 = 1101 – 1080 = 21.

E = 1080; Pass 21

Green Relay Seat A

Perpendicular form for 5x – 3y – 4 = 0 is 3x+ 5y. 3x + 5y = 2x + ay: $\frac{3}{5} = \frac{2}{a}$, 3a = 10. So a = 10/3. Pass back: 15A = 15(10/3) = 50. **A = 10/3; Pass 50**

Green Relay Seat B

From Blue Seat B: Jones's traveled 80(2½) = 200. Pass back: 8(B + X) – 44 = 8(200 + 50) – 44 2000 – 44 = 1956. **B = 200; Pass 1956**

Green Relay Seat C

$r^2 + 15^2 = 17^2$, r = 8. Area = 64π. Pass back: $X - \frac{C}{\pi} = 1956 - \frac{64\pi}{\pi}$ = 1956 – 64 = 1892.

C = 64π; Pass 1892

Green Relay Seat D

5! = 120, 6! = 720. The 4th term is 120/(720/120) = 120/6 = 20. Pass: $\frac{X}{4} - D = \frac{1892}{4} - 20$ = 473 – 20 = 453. **D = 20; Pass 453**

Green Relay Seat E

There should be $4 \cdot 5 \cdot 5 \cdot 5 \cdot 5$ = 100(25) = 2500 numbers. Pass in: E – 5X = 2500 – 5(453) = 2500 – 2265 = 235. **E = 2500; Pass 235**

Pink Relay Seat A

(1) $\frac{a+b}{2} = x$, (2) $\frac{b+x}{2} = \frac{a+b+1}{2}$. In (1): a + b = 2x. In (2): b + x = a + b + 1 or x = a + 1. Subbing into (1): a + b = 2(a + 1), so a – b = -2. Pass: 8 – A = 8 – (-2) = 10. **A = -2; Pass 10**

Pink Relay Seat B

Let Alex's age = x + 10, Ben's = x. Then 2/3(x + 10 + 12) = x + 12 ➔ 2x + 44 = 3x + 36 ➔ 8 = x. So Alex is 18. Pass back: B – X = 18 – 10 = 8. **B = 18; Pass 8**

Pink Relay Seat C

If diagonals differ by 6, then ½ the diagonals differ by 3. Thus $x^2 + (x + 3)^2 = 225$.

$x^2 + x^2 + 6x + 9 = 225$ ➔ $2x^2 + 6x - 216 = 0$ ➔ $x^2 + 3x - 108 = 0$ ➔ (x + 12)(x – 9) = 0. x = 9

The diagonals are 18 and 24. Area = 1/2(18)(24) = 216. Pass: $\frac{C}{X} = \frac{216}{8}$ = 27. **C = 216; Pass 27**

Pink Relay Seat D

$\frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{2 \cdot 3 \cdot 2}$ = 10. Pass: DX = 10(27) = 270. **D = 10; Pass 270**

Pink Relay Seat E

$\frac{2^{14x+11}}{8} \cdot \frac{4^{21x-1}}{32} = \frac{8^{7x+10}}{128}$ ➔ $2^{14x+8} \cdot 2^{42x-7} = 2^{21x+23}$ ➔ 56x + 1 = 21x + 23 ➔ 35x = 22

So x = 22/35. Pass in: 7EX = 7($\frac{22}{35}$)(270) = 22(54) = 1188. **E = 22/35; Pass 1188**

Yellow Relay Seat A

$\frac{a+b}{2} = x$, so (1) a + b = 2x. $\frac{b+x}{2} = \frac{a+b-2}{2}$, so b + x = a + b – 2 or (2) x = a – 2. Subbing in to (1): a + b = 2(a – 2) = 2a – 4 ➔ a – b = 4. Pass: 8 – A = 8 – 4 = 4. **A = 4; Pass 4**

Yellow Relay Seat B

Refer to Pink Seat A, Ben is 8. Pass back: B – X = 8 – 4 = 4. **B = 8; Pass 4**

Yellow Relay Seat C

$d^2 + (d + 2)^2 = 100$ ➔ $2d^2 + 4d + 4 = 100$ ➔ $d^2 + 2d - 48 = 0$ ➔ (d + 8)(d – 6) = 0, so d = 6.

The diagonals are 12 and 16. Area = 1/2(12)(16) = 96. Pass: $\frac{C}{X} = \frac{96}{4}$ = 24. **C = 96; Pass 24**

Yellow Relay Seat D

$\frac{5!}{1!4!}$ = 5. Pass back: DX = (5)(24) = 120. **D = 5; Pass 120**

Yellow Relay Seat E

$\frac{2^{14x+11}}{\sqrt{2}} \cdot \frac{4^{21x-1}}{32} = \frac{8^{7x+10}}{128}$ = $2^{14x+11-1/2} \cdot 2^{42x-2-5} = 2^{21x+30-7}$ ➔ 56x + 3½ = 21x + 23 ➔

35x = 19½ ➔ 35x = $\frac{39}{2}$ ➔ x = $\frac{39}{2} \cdot \frac{1}{35} = \frac{39}{70}$. Pass: (X + 20)E = (120 + 20)$\left(\frac{39}{70}\right)$ = 78.

E = $\frac{39}{70}$; Pass 78

Answer Sheet   States 2016

Round 1 Individuals

1. **80**
2. $\pm\sqrt{10}$, $\pm 1/10$
3. **31**

Round 2 Individuals

1. **4**
2. **87**
3. $\pi/6$

Round 3 Individuals

1. **$1.10**
2. **8**
3. **131**

Round 4 Individuals

1. **240**
2. **4**
3. $30^\circ, 150^\circ, 270^\circ$

Round 5 Individuals

1. **38**
2. 36π
3. **2**

Round 6 Individuals

1. **100**
2. **3**
3. **12**

Round 1 Team

1. **1/4** or **.25** or **25%**
2. **1/5**
3. **-1**
4. **25** or **25 ft**
5. $3\pi/2$
6. $\frac{3-\sqrt{3}}{2}$
7. **35/16**
8. **15/2** or **7.5**

Round 2 Team

1. **1/4** or **.25** or **25%**
2. **-2/r**
3. $\pm 9, \pm 16$
4. $2\sqrt{7}$
5. **12:40 pm**
6. **5**
7. $2\sqrt[3]{2}$
8. **500,050**

	Blue Relay		Green Relay		Pink Relay		Yellow Relay	
Seat	**Ans.**	**Pass**	**Ans.**	**Pass**	**Ans.**	**Pass**	**Ans.**	**Pass**
A	6/5	18	10/3	50	-2	10	4	4
B	130	1997	200	1956	18	8	8	4
C	49π	1948	64π	1892	216	27	96	24
D	120	367	20	453	10	270	5	120
E	1080	21	2500	235	22/35	1188	39/70	78

Individuals Round 1

1 80

2 $\pm\sqrt{10}, \pm\frac{1}{10}$ **or** $\pm 10^{½}, \pm 0.1$

3 31

Individuals Round 2

1 4 **or** 4 u^2 **or** 4 un^2 **or** 4 $units^2$

2 87 **or** ± 87

3 $\frac{\pi}{6}$ **or** $\pi : 6$ **or** π *to* 6

Individuals Round 3

1 $1.10

1.10

1 *dollar* 10 *cents*

1 *dollar and* 10 *cents*

2 8

3 131

Individuals Round 4

1 240

2 4

3 $30°$, $150°$, $270°$ **or** 30, 150, 270

Individuals Round 5

1 38

2 36π

3 2

Individuals Round 6

1 100

2 3

3 12

Relay Round 1

	Blue Relay	Green Relay
A	$\frac{6}{5}$ **or** 1.2 **or** $1\frac{1}{5}$	$\frac{10}{3}$ **or** $3.\overline{3}$ **or** $3\frac{1}{3}$
B	130	200
C	49π	64π
D	120	20
E	1080	2500
P	21	235

Relay Round 2

	Pink Relay	Yellow Relay
A	-2	4
B	18	8
C	216	96
D	10	5
E	$\frac{22}{35}$	$\frac{39}{70}$
P	1188	78

Team Round 1

1 ¼ or .25 or 25%

2 ⅕

3 -1

4 25 or 25ft

5 $\frac{3\pi}{2}$

6 $\frac{3-\sqrt{3}}{2}$ or $\frac{\sqrt{3}-3}{-2}$

7 $\frac{35}{16}$ or 2.1875 or $2\frac{3}{16}$

8 $\frac{15}{2}$ or 7.5 or 7½

Team Round 2

1 ¼ **or** .25 **or** 25%

2 $\frac{-2}{r}$

3 $\pm 9, \pm 16$ **or**
$\pm 9 \; and \; \pm 16$

4 $2\sqrt{7}$

5 **12:40pm**

6 5

7 $2\sqrt[3]{2}$

8 500050