Individuals Round 1 States – 2013

3 pts 1. Find the slope-intercept form of the equation of the line passing through the points (-2, -7) and (5, -21).

Ans. ______________________

4 pts 2. Find the smallest sum A + B + C + D, if all are natural numbers and

$$X^{3/2}Y^{2/5}Z^{1/3} = \sqrt[A]{X^B Y^C Z^D}$$

Ans. ________________

5 pts 3. A pair of dice are tossed. Find the probability that the sum of the numbers on the top face is a prime number.

Ans. ________________

Individuals Round 2 States – 2013

3 pts 1. Ernie has 20 pieces of chewing gum. If he gives Frank 20%, George 30% and Henry 10%, how many pieces does he have left for himself?

Ans. ___________________

4 pts 2. If A = 2 + 3*i* and B = 6 – 5*i*, where $i = \sqrt{-1}$, find the value of $A^2 + 2AB + B^2$.

Ans. ________________________

5 pts 3. Each of the following data points x, x + 5, *n*, x + 2, x – 1, and x – 2 are natural numbers, where *n* is the second largest data point. The mean is one unit larger than the median. The product of the smallest data point and *n* exceeds the mean by 4. Find the mean.

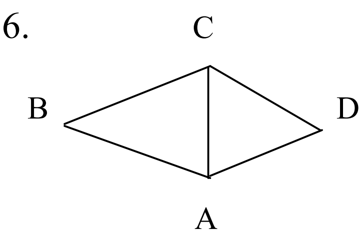
Ans. ________________________

Individuals Round 3 States – 2013

3 pts 1. Find the LCM of 48 and 64.

Ans. ________________

4 pts 2. ΔABC is similar to ΔACD, AB = 4, BC = 6 and AC = 6. Find the perimeter of ΔACD.



Ans. ____________________

5 pts 3. A panel, composed of members of 3 political parties, has 60 Democrats, 64 Republicans, and 8 Greens. A committee is chosen from the panel that is proportional to the party members and relatively prime. If a two-person leadership team is chosen randomly from the committee, how many different pairs could be selected such that both members of the team are from the same party?

Ans. ________________

Individuals Round 4 States 2013

3 pts 1. If a * b = $a^2 - \frac{b}{2}$, find the value of 2 * (3 * 4) .

Ans. __________________

4 pts 2. The area of ΔABC = 90. AC is five times as long as AB. $m\angle A = 30^o$. Find the sum of the lengths of sides AC and AB.

Ans. ______________________

5 pts 3. Find the smallest sum N + M such that M and N are natural numbers which satisfy the equation:

$$\frac{N}{M}+\frac{M}{3N}=\frac{2M+3}{6N}-\frac{2N-M}{4MN}$$

Ans. ______________________

Individuals Round 5 States 2013

3 pts 1. Determine the number of degrees in each angle of a regular octagon.

Ans. ______________________

4 pts 2. Find the middle term of the expansion of $\left(2x+\frac{1}{4}y^2\right)^6$.

Ans. ______________________

5 pts 3. The sums of the two infinite decreasing series A and B are the same. The sum of the first term of the series A and the first term of series B is 700. The sum of the common ratios is .6. One common ratio is twice the other. Find the sum of either series.

Ans. ______________________

Individuals Round 6 States 2013

3 pts 1. The measure of one of the angles of an isosceles triangle is 70^o. What are the possible measures of the other two angles?

Ans. ____________________________

4 pts 2. Find the determinant of the matrix: $\begin{bmatrix} 2 & 4 & -1 \\ 3 & -2 & 2 \\ 1 & 3 & 4 \end{bmatrix}$

Ans. ______________________

5 pts 3. Find all values of θ, where $0^o \le \theta < 360^o$, such that

$$2\sin\theta - 2\sin\theta\cos\theta + \cos\theta = 1.$$

Ans. ____________________________

Round 1   Team   States 2013

4 pts 1. Find the sum of the x-intercept, y-intercept and slope of the line whose equation is $3x + 4y = 7$. Express your answer as a mixed number.

(1) **Ans.** ________________**4pts**

4 pts 2. The original price of a Kindle is $100. A 10% discount of the original price was issued after the first week. A 9% discount of the original price was issued after the second week. This discount continued weekly by a difference of 1% until there was no discount. What was the price of the Kindle at that time?

(2) **Ans.** ________________**4pts**

6 pts 3. Simplify: $\dfrac{X^3 - 6X^2 + 11X - 6}{X^3 - 2X^2 - 5X + 6}$.

(3)**Ans.** ________________**6pts**

6 pts 4. Find the quotient when 22024 base six is divided by 52 base six. Give the answer in base six.

(4)**Ans.** ________________**6pts**

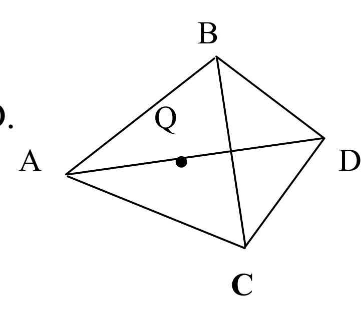
6 pts 5. $f(X) = 2X - 3$ and $g(X) = \dfrac{1 - X^2}{X}$. For what values of X is the domain of $\dfrac{g(X)}{f(X)} > 0$?

(5)**Ans** ________________________________**6pts**

8 pts 6. Lines $y = 2x + 1$ and $y = -2x + 11$ intersect at point P. $y = a$ contains the base of triangle PQR where $y = a$ is below P and points R and Q are on $y = a$. Find R and Q if the area of triangle PQR is 18.

(6) ________________________**8pts**

8 pts 7. Triangle ABC is equilateral with perimeter 36.
Q is the centroid of ΔABC and the midpoint of segment AD.
Find the perimeter of quadrilateral ABDC.



(7) ________________**8pts**

8 pts 8. Find the largest value of f(x), such that f(x) is a real number and
$f(x) = \sqrt{8x - x^2} - \sqrt{14x - x^2 - 48}$

(8)**Ans**____________________**8pts**

Team Round 2 States 2013

4 pts 1. Find the sum of the coefficients when $3x^2 + 4xy - 10y^2 - 7y^3$ is subtracted from $3xy - 7y^2 - y^3 + x$.

(1)**Ans** ______________________________**4pts**

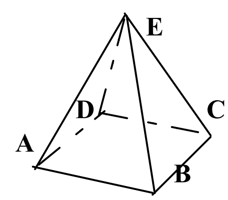
4 pts 2. Find the base ten value of 5t63 base twelve.

(2)**Ans**_____________________**4pts**

6 pts 3. Simplify $\dfrac{1}{1+\dfrac{1}{x-\dfrac{1}{1-x}}}$

(3)**Ans**____________________________**6pts**

6 pts 4. ABCDE is a right square pyramid. AC = 6 and AE = 5. Find the volume of the pyramid.



(4)**Ans**____________________**6pts**

6 pts 5. Write the equation of the line in slope-intercept form which connects the vertices of the parabolas $y = 3x^2 - 6x + 1$ and $y = -2x^2 - 8x + 11$.

(5)**Ans** ___________________________**6pts**

8 pts 6. Find $|a+b|$, if $\dfrac{a-bi}{b-ai} = \dfrac{12-5i}{13}$.

(6)**Ans**_____________________**8pts**

8 pts 7. Find all values of x such that $\dfrac{|3x-1|}{|x+1|} \le |x-1|$.

(7)**Ans**____**8pts**

8 pts 8. Let $a_1, a_2, a_3, \cdots, a_k$ be an infinite arithmetic series. $a_4 + a_7 + a_{10} = 17$ and $a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} = 77$. If $a_k = 13$, find *k*.

(8)**Ans**_________________**8pts**

Seat A   Blue Relay   States 2013

Find the greatest possible integer x, such that $3x + 4 > 5x - 2$.

Pass back: 4^A A = Your answer

Seat B   Blue Relay   States 2013

Joseph's age is 9 years less than 3 times Mary's age. In 13 years, Mary's age will be 4/7 of what Joseph's age will be then. How old is Mary now?

Pass back: $8(X - B)$ B = Your answer.   X = TNYWR

Seat C   Blue Relay   States 2013

A square is inscribed in a circle of radius 8. The area inside the circle and outside the square is $A\pi - B$. Find $B - A$.

Pass back: $C - 6X$.   C = Your answer.   X = TNYWR

Seat D   Blue Relay   States 2013

Determine the largest possible value of X, such that $\frac{X}{X+1} + \frac{3X-1}{3X} = \frac{X+1}{X}$.

Pass back: $\frac{X}{2D}$ D = Your answer   X = TNYWR

Seat E   Blue Relay   States 2013

Find the largest value of x such that: $\log_4(2x^2 - 45) - \log_4(3x - 9) = \log_4(x - 3)$

Pass in: $4E - 5X$ E = Your answer.   X = TNYWR

Seat A Green Relay States 2013

Find the greatest possible integer x, such that $2x - 5 > 6x + 7$.

Pass back: $A + 5$. A = Your answer.

Seat B Green Relay States 2013

Joseph's age is 9 years less than 3 times Mary's age. In 13 years, Mary's age will be 4/7 of what Joseph's age will be then. How old is Joseph now?

Pass back: $B/(X + 3)$ B = Your answer. X = TNYWR

Seat C Green Relay States 2013

A square is inscribed in a circle of radius 8. The area inside the circle and outside the square is $A\pi - B$. Find $A + B$.

Pass back: $C - 20X$ C = Your answer. X = TNYWR

Seat D Green Relay States 2013

Determine the largest possible value of X, such that $\frac{X}{X+1} + \frac{2X+1}{4X} = \frac{X+4}{4}$.

Pass back: $X - D^3$ D = Your answer. X = TNYWR

Seat E Green Relay States 2013

Find the smallest value of x such that: $\log_4(2x^2 - 45) - \log_4(3x - 9) = \log_4(x - 3)$

Pass in: $6E - 3X$. E = Your answer. X = TNYWR

Seat A Pink Relay States 2013

Simplify: $\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{2+1}}}$ Pass back: $\dfrac{A}{(-1)^3}$ A = Your answer.

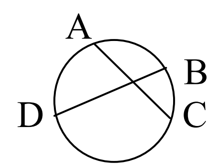
Seat B Pink Relay States 2013

The sum of three consecutive positive integers is three less than the product of the smallest and the largest of these integers. Find the value of the sum of the three integers.

Pass back: B/(X – 2) B = Your answer. X = TNYWR

Seat C Pink Relay States 2013

G is the midpoint of $\overline{AC}$, DG = 4BG, and BD = 10. Find the length of $\overline{AC}$.



Pass back: $X^{C/2}$ C = Your answer. X = TNYWR

Seat D Pink Relay States 2013

Find the sum of the coordinates of the ordered pairs of the intersection(s) of
$x + 4y = 1$ and $y = x^2 + x - 5$.

Pass back: DX D = Your answer. X = TNYWR.

Seat E Pink Relay States 2013

The endpoints of the major axis of an ellipse are (5, -13) and (5, 7). A focus is (5, 3). The equation of the ellipse is $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-h)^2}{b^2}$. Find the value of $h + k + a + b$.

Pass in: E/(X + 9) E = Your answer. X = TNYWR

Yellow Relay Seat A

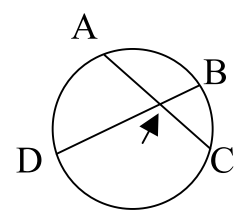
Simplify: $\dfrac{1}{1-\dfrac{1}{1+\dfrac{1}{3+1}}}$ Pass back: $\dfrac{4}{3-A}$ A = Your answer.

Yellow Relay Seat B

The sum of three consecutive positive integers is 5 less than the product of the two smallest of these integers. Find the sum of the three integers.

Pas Back: B/(X-3) B = your answer. X = TNYWR

Yellow Relay Seat C



G is the midpoint of $\overline{AC}$. DG = 4BG. BD = 15.

Find the length of $\overline{AC}$.

Pass back: $(X + 1)^{C/2}$ C = Your answer. X = TNYWR

Yellow Relay Seat D

Find the sum of the coordinates of the ordered pairs of the intersection(s) of

$x + 3y = 1$ and $y = x^2 + x - 1$.

Pass back: $\dfrac{-X}{3D}$ D = Your answer. X = TNYWR

Yellow Relay Seat E

The endpoints of the major axis of an ellipse are (5, -13) and (5, 7). A focus is (5, 3). The equation of the ellipse is $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-h)^2}{b^2}$. Find the value of $(a + b) - (h + k)$.

Pass in: X/(E – 4) E = Your answer. X = TNYWR

Solutions – Individuals Round 1

1. Slope: $\frac{-7-(-21)}{-2-(5)} = \frac{14}{-7} = -2$. y = -2x + b ➔ -7 = -2(-2) + b. b = -11. **Ans. y = -2x – 11.**

2. $X^{3/2}Y^{2/5}Z^{1/3} = X^{45/30}Y^{12/30}Z^{10/30} = \sqrt[30]{X^{45}Y^{12}Z^{10}}$. 30 + 45 + 12 + 10 = 97. **Ans. 97**

3. There is 1 way to get a 2, 2 to get a 3, 4 to get a 5, 6 to get a 7, 2 to get an 11. This makes 15/36 = 5/12. **Ans. 5/12**

Individuals Round 2

1. 40% left. .4(20) = 8. **Ans. 8**

2. $A^2 + 2AB + B^2 = (A + B)^2$. $(8 - 2i)^2 = 64 - 32i + 4i^2 = 64 - 32i - 4$. **Ans. 60 – 32*i***

3. Median = x + 1, mean = $\frac{5x+6+n}{6}$. $\frac{5x+6+n}{6} - 1 = x + 1$ ➔ 5x + 6 + n – 6 = 6x + 6 ➔ n = x + 6. (x – 2)(x + 6) = (x + 2) + 4 ➔ $x^2 + 4x - 12 = x + 6$ ➔ $x^2 + 3x - 18 = 0$ ➔ (x + 6)(x – 3) = 0. Thus x = 3 and the mean is x + 2 = 5. **Ans. 5**

Individuals Round 3

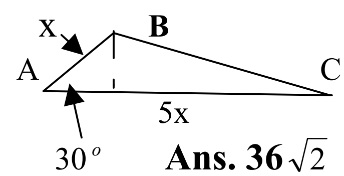
1. 48 = $2\cdot2\cdot2\cdot2\cdot3$, 64 = $2\cdot2\cdot2\cdot2\cdot2\cdot2$. LCM = $(2\cdot2\cdot2\cdot2)(3\cdot2\cdot2)$ = 16(12) = 192 **Ans. 192**

2. $\frac{AB}{AC} = \frac{AC}{AD}$ ➔ $\frac{4}{6} = \frac{6}{x}$, so x = 9. Perimeter = 6 + 9 + 9 = 24. **Ans. 24**

3. 60:64:8 = 15:16:2. $\binom{15}{2} + \binom{16}{2} + \binom{2}{2} = \frac{15\cdot14}{2} + \frac{16\cdot15}{2} + \frac{2\cdot1}{2}$ = 15(15) + 1 = 226. **Ans. 226**

Individuals Round 4

1. 3 * 4 = $3^2 - \frac{4}{2} = 7$. 2 * 7 = $2^2 - \frac{7}{2} = \frac{1}{2}$ **Ans. 1/2**



2. In the figure, drop a perpendicular from B to side AC. The length of the perpendicular is ½ x. So the area of the triangle is ½ (½ x)(5x) = 90 ➔ $5x^2 = 360$, so x = $\sqrt{72} = 6\sqrt{2}$. So 6x = $36\sqrt{2}$. **Ans. 36$\sqrt{2}$**

3. $\frac{N}{M} + \frac{M}{3N} = \frac{2M+3}{6N} - \frac{2N-M}{4MN}$ ➔ $12N^2 + 4M^2 = 4M^2 + 6M - 6N + 3M$ ➔ $12N^2 + 6N - 9M = 0$
$4N^2 + 2N = 3M$ ➔ $2N^2 + N = \frac{3}{2}M$. The smallest natural number for *M* is 2. Thus $2N^2 + N - 3 = 0$ or (2*N* + 3)(*N* – 1) = 0. So *N* = 1, and *N* + *M* = 3. **Ans. 3**

Individuals Round 5

1. $180 - \frac{360}{8} = 180 - 45 = 135$. **Ans. 135**

2. A 6th power polynomial has 7 terms, the 4th being the middle term, thus

$$\binom{6}{3}(2x)^3\left(\frac{1}{4}y^2\right)^3 = 20\left(8x^3\right)\left(\frac{1}{64}y^6\right) = \frac{5}{2}x^3y^6.$$ **Ans.** $\frac{5}{2}x^3y^6$

3. $\frac{700-x}{1-a} = \frac{x}{1-b}$. b = 2a, and a + b = .6 ➔ a + 2a = .6, so a = .2, thus b = .4. Using the original equation: $\frac{700-x}{.8} = \frac{x}{.6}$ ➔ 420 - .6x = .8x ➔ 420 = .14x, thus x = 300. $\frac{300}{.6} = 500$.

Ans. 500

Individuals Round 6

1. If the 70^o angle is the vertex angle, the other two are equal and both are 55^o. If the 70^o angle is one of the base angles, the other two are 70^o and 40^o. **Ans. 55^o, 55^o or 70^o, 40^o**

2. $\begin{bmatrix} 2 & 4 & -1 \\ 3 & -2 & 2 \\ 1 & 3 & 4 \end{bmatrix}$ = (-2 – 12 – 48) – (-16 + 8 – 9) = - 62 + (-17) = - 79. **Ans. -79**

3. $2\sin\theta - 2\sin\theta\cos\theta + \cos\theta = 1$ ➔ $2\sin\theta - 1 - 2\sin\theta\cos\theta + \cos\theta = 0$ ➔ $2\sin\theta - 1 - \cos\theta(2\sin\theta - 1) = 0$ ➔ $(1-\cos\theta)(2\sin\theta - 1) = 0$. So $\cos\theta = 1$ or $\sin\theta = 1/2$. $\cos\theta = 1$ at 0^o. $\sin\theta = 1/2$ at 30^o and 150^o. **Ans. 0^o, 30^o, 150^o**

Team Round 1

1. 3x + 4y = 7. x-int. = 7/3 = 2 ⅓, y – int. = 7/4 = 1¾ , slope = -3/4. Sum = 3 ⅓. **Ans.** 3 ⅓

2. The sum of the percents is 55%. The rest is 45%. 45% of 100 = 45. **Ans. $45**

3. $$\frac{X^3 - 6X^2 + 11X - 6}{X^3 - 2X^2 - 5X + 6} = \frac{(X-1)(X-2)(X-3)}{(X-1)(X+2)(X-3)} = \frac{x-2}{x+2}$$ **Ans.** $\frac{x-2}{x+2}$

4. $52_6 \overline{)\,22024_6}$ with quotient 235_6

```
        235
52 ) 22024
     144
      322
      240
       424
       424
```

Ans. 235_6

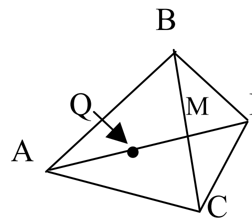
5. $\dfrac{g(x)}{f(x)} = \dfrac{\frac{1-x^2}{x}}{2x-3} = \dfrac{1-x^2}{x(2x-3)} > 0$. Two of the critical points are 1½ and 0,which make the denominator zero. $1 - x^2 = 0$ produces two more, 1 and -1. -----o▬▬o-----o▬▬o-----

Plugging into the last fraction for positive results: -1 0 1 1½

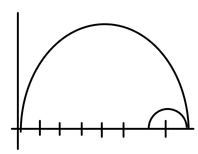
-2 => $\dfrac{-}{-\cdot-} = -$; - ½ => $\dfrac{+}{-\cdot-} = +$; 1¼ => $\dfrac{-}{+\cdot-} = +$; 2 => $\dfrac{-}{+\cdot+} = -$

Ans. -1 < x < 0 or 1 < x < 1½

6. (1) y = 2x + 1, (2) y = -2x + 11. Solving for P: (1) + (2): 2y = 12, so y = 6. In (1): 6 = 2x + 1, so x = 2½. P = (2½ , 6). Since the slope is 2 which is the ratio of rise to run, then the height and the base of the triangle are equal. ½ h^2 = 18, so the height is 6 and the base is 6. If you drop down 6 from P(2½, 6) you end up on the x-axis at (2½, 0). R and Q are 3 units horizontally from (2½, 0) = (2½ ± 3, 0) **Ans. (- 1/2, 0), (5 ½ , 0)**



7. The centroid is 2/3 of the distance from the vertex to the base. Since the perimeter is 36 then each side of the triangle is 12. Thus CM = 6 and AM = $6\sqrt{3}$, and AQ = $4\sqrt{3}$ and QM = $2\sqrt{3}$ and DM = $2\sqrt{3}$. Since MC = 6, then DC = $\sqrt{6^2 + (2\sqrt{3})^2}$ = $\sqrt{36+12} = \sqrt{48} = 4\sqrt{3}$. **Ans. 24 + 8$\sqrt{3}$**



8. $\sqrt{8x - x^2}$ is the top half of a circle with center at (4, 0) and radius 4. $\sqrt{14x - x^2 - 48}$ is the top half of a circle with center at (7, 0) and radius 1. Real values of x can only exist from 6 to 8, the largest sum being at x = 6. f(6) = $\sqrt{48-36}$ = $\sqrt{12}$ = $2\sqrt{3}$. **Ans. 2$\sqrt{3}$**

Team Round 2

1. $3xy - 7y^2 - y^3 + x - (3x^2 + 4xy - 10y^2 - 7y^3) = -xy + 3y^2 + 6y^3 + x - 3x^2$. **Ans. 6**

2. $5t63_{12}$ = 5(1728) + 10(144) + 6(12) + 3 = 8640 + 1440 + 72 + 3 = 10,155. **Ans. 10,155**

3. $\dfrac{1}{1+\dfrac{1}{\dfrac{x(1-x)-1}{1-x}}} = \dfrac{1}{1+\dfrac{1}{\dfrac{x-x^2-1}{1-x}}} = \dfrac{1}{1+\dfrac{1-x}{x-x^2-1}} = \dfrac{1}{1+\dfrac{x-1}{x^2-x+1}} = \dfrac{1}{\dfrac{x^2-x+1+x-1}{x^2-x+1}}$ **Ans.** $\dfrac{x^2-x+1}{x^2}$

4. The area of the base is ½ $d_1 d_2$ = ½ (6)(6) = 18. Dropping the altitude from E to the base hits the midpoint of $\overline{AC}$. The altitude = $\sqrt{5^2 - 3^2}$ = 4. Vol. = $\frac{1}{3}(18)4 = 24$. **Ans. 24**

5. The x coordinate of the vertex of each parabola is $\frac{-b}{2a}$. For y = $3x^2 - 6x + 1$, the x coordinate is $\frac{-(-6)}{2(3)} = 1$. Its vertex is (1, -2). For y = $-2x^2 - 8x + 11$, the x coordinate is $\frac{-(-8)}{2(-2)} = -2$. Its vertex is (-2, 19). Slope = $\frac{-2-19}{1+2} = -7$ ➔ y = -7x + b ➔ -2 = -7(1) + b. So b = 5. **Ans. y = -7x + 5**

6. $\frac{a-bi}{b-ai}\cdot\frac{b+ai}{b+ai} = \frac{ab-b^2i+ab+a^2i}{b^2-a^2i^2} = \frac{2ab+(a^2-b^2)i}{a^2+b^2} = \frac{12-5i}{13}$. Thus (1) $a^2-b^2=-5$, (2) $2ab=12$, and (3) $a^2+b^2=13$. (1) + (3): $2a^2 = 8$, so a = ± 2. In (2): if a = 2, then b = 3; If a = -2, then b = -3. Thus $|a+b| = 5$. **Ans. 5**

7. x = -1 is a critical point for $\frac{|3x-1|}{|x+1|} \le |x-1|$, since it makes the denominator 0. Multiplying by $|x+1|$,we get $|3x-1| \le |(x^2-1)|$. The other critical points are found when $\pm(3x-1) = x^2 - 1$ ➔ (1) $3x - 1 = x^2 - 1$ or (2) $1 - 3x = x^2 - 1$. In (1): $x^2 - 3x = 0$ ➔ x(x – 3) = 0. So x = 0 or 3. In (2): $x^2 + 3x - 2 = 0$. Since this does not factor, quad. Equation: $x = \frac{-3\pm\sqrt{9-4(-2)}}{2} = \frac{-3\pm\sqrt{17}}{2}$. Since $\sqrt{17}$ is a little more than 4, we will use approximate critical points of: $\frac{-3+4}{2} = \frac{1}{2}$ and $\frac{-3-4}{2} = -3\frac{1}{2}$. Plugging interval points into original:

◄o------o------o━━━o------o►

-3½ -1 0 ½ 3

-4 => $\frac{13}{3} < 5$, yes; -2 => $\frac{7}{1} < 3$, no; - ½ => $\frac{4\frac{1}{2}}{\frac{1}{2}} < 1\frac{1}{2}$, no; ¼ => $\frac{\frac{1}{4}}{\frac{5}{4}} < \frac{3}{4}$, yes

2 => $\frac{5}{3} < 1$, no; 4 => $\frac{11}{5} < 3$, yes. **Ans.** $x \le \frac{-3-\sqrt{17}}{2}$ **or** $0 \le x \le \frac{-3+\sqrt{17}}{2}$ **or** $x \ge 3$

8. $a_4 + a_7 + a_{10} = 17$ is an arithmetic sequence and the middle term a_7 must = 17/3. Likewise $a_4 + a_5 + a_6 + a_7 + a_8 + a_9 + a_{10} + a_{11} + a_{12} + a_{13} + a_{14} = 77$ also produces middle term a_9 = **77**/11 = 7. $a_7 + 2d = a_9$, where d is the common difference. Thus 17/3 + 2d = 7 ➔ 2d = 4/3, so d = 2/3. $a_9 = 7 = a_1 + 8(2/3)$, so $a_1 = 1\frac{2}{3}$. If $a_k = 13$, then $13 = 1\frac{2}{3} + (k-1)2/3$ ➔ $39 = 5 + (k-1)2$ ➔ $17 = k - 1$, so $k = 18$. **Ans. 18**

Blue Relay Seat A

3x + 4 > 5x – 2 ➔ 6 >2x, so x < 3. Greatest integer is 2. Pass: $4^A = 4^2 = 16$.

Ans. A = 2, Pass: 16

Blue Relay Seat B

Let M = Mary's age, Joseph's age = 3M – 9. $\frac{4}{7}(3M - 9 + 13) = M + 13$ ➔

4(3M + 4) = 7(M + 13) ➔ 12M + 16 = 7M + 91 ➔ 5M = 75, so M = 15. Mary is 15, Joseph is 36. Pass: 8(X – B) = 8(16 – 15) = 8. **Ans. B = 15, Pass: 8**

Blue Relay Seat C

Area of circle is 64π. Area of square is ½ (16)(16) = 128. Area of region 64π - 128 = $A\pi$ - B, so B – A = 64. Pass: C – 6X = 64 – 6(8) = 16. **Ans. C = 64, Pass: 16**

Blue Relay Seat D

$\frac{X}{X+1}+\frac{3X-1}{3X}=\frac{X+1}{X}$ ➔ $3x^2 + (3x – 1)(x + 1) = 3(x + 1)^2$ ➔

$3x^2 + 3x^2 + 2x – 1 = 3x^2 + 6x + 3$ ➔ $3x^2 - 4x – 4 = 0$ ➔ (3x + 2)(x – 2) = 0, so x = 2.

Pass: $\frac{X}{2E} = \frac{16}{2(2)} = 4$. **Ans. D = 2, Pass: 4**

Blue Relay Seat E

$\log_4(2x^2 - 45) - \log_4(3x – 9) = \log_4(x – 3)$ ➔ $\frac{2x^2-45}{3x-9}=x-3$ ➔ $2x^2 - 45 = 3x^2 - 18x + 27$

$0 = x^2 - 18x + 72$ ➔ 0 = (x – 6)(x – 12). Thus E = 12. Pass in: 4E – 5X = 4(12) – 5(4) = 28. **Ans. E = 12, Pass: 28**

Green Relay Seat A

2x – 5 > 6x + 7 ➔ -12 > 4x, so x < 3. Greatest integer is -4. Pass: A + 5 = -4 + 5 = 1. **Ans. A = -4, Pass: 1**

Green Relay Seat B

From Blue Relay B, Joseph is 36. Pass: B(X + 3) = 36(1 + 3) = 9. **Ans. B = 36, Pass: 9**

Green Relay Seat C

From Blue C, A + B = 192. Pass: C – 20X 192 – 20(9) = 12. **Ans. C = 192, Pass: 12**

Green Relay Seat D

$\frac{X}{X+1}+\frac{2X+1}{4X}=\frac{X+4}{4}$ ➔ $4x^2 + (2x + 1)(x + 1) = x(x + 1)(x + 4)$ ➔

$4x^2 + 2x^2 + 3x + 1 = x^3 + 5x^2 + 4x$ ➔ $0 = x^3 - x^2 + x – 1 = x^2(x – 1) + (x – 1)$ ➔

$(x – 1)(x^2 - 1) = 0 = (x – 1)(x – 1)(x + 1) = 0$. Largest value is 1.

Pass: $X - D^3 = 12 – (1)^3 = 11$. **Ans. D = 1, Pass: 11**

Green Relay Seat E

From Blue E: E = 6. Pass in: 6E – 3X = 6(6) – 3(11) = 3. **Ans. E = 6, Pass: 3**

Pink Relay Seat A

$\frac{1}{1-\frac{1}{1+\frac{1}{2+1}}} = \frac{1}{1-\frac{1}{\frac{4}{3}}} = \frac{1}{1-\frac{3}{4}} = \frac{1}{\frac{1}{4}} = 4$. Pass: $\frac{A}{(-1)^3} = -4$. **Ans: A = 4, Pass: -4**

Pink Relay Seat B

Let the integers be x – 1, x, x + 1: $3x = x^2 - 1 - 3$ ➔ $0 = x^2 - 3x - 4$ ➔ $(x - 4)(x + 1) = 0$
x = 4. Integers: 3, 4, 5. Sum: 12. Pass: B/(X – 2) = 12/(-4 – 2) = -2. **Ans. B – 12, Pass: -2**

Pink Relay Seat C

BG = 2, GD = 8. $AG^2 = 16$, so AC = 8. Pass: $X^{C/2} = (-2)^{8/2} = 16$. **Ans. C = 8, Pass: 16**

Pink Relay Seat D

(1) x + 4y = 1, (2) $y = x^2 + x - 5$. In (1): x = 1 – 4y. Subbing into (2):
$y = (1 - 4y)^2 + (1 - 4y) - 5$ ➔ $y = 1 - 8y + 16y^2 - 4y - 4$ ➔ $0 = 16y^2 - 13y - 3$.
(16y + 3)(y – 1) = 0. So y = 1 or -3/16. If y = 1, then x = -3. If y = -3/16, then x = 1¾.
Adding all coordinates: 1 + (-3) + (-3/16) + 1¾ = -7/16. Pass: DX = (-7/16)(16) = -7.

Ans. D = -7/16, Pass: -7

Pink Relay Seat E

Since (5, -13) and (5, 7) are endpoints of major axis then center is (5, -3) and semi-major axis is 10 units long. The focus (5, 3) is 6 units from the center, so we have a 6-8-10 triangle making the semi-minor axis 8 units long. Thus the equation
$\frac{(x-5)^2}{8^2} + \frac{(y+3)^2}{10^2} = 1$. h + k + a + b = 5 + (-3) + 8 + 10 = 20. Pass: $\frac{E}{X+9} = \frac{20}{-7+9} = 10$.

Ans. E = 20, Pass: 10

Yellow Relay Seat A

$\frac{1}{1-\frac{1}{1+\frac{1}{3+1}}} = \frac{1}{1-\frac{1}{\frac{5}{4}}} = \frac{1}{1-\frac{4}{5}} = 5$. Pass: $\frac{4}{3-A} = \frac{4}{3-5} = -2$. **Ans. A = 5, Pass: -2**

Yellow Relay Seat B

Let integers be x, x + 1, x + 2. $3x + 3 = x^2 + x - 5$ ➔ $0 = x^2 - 2x - 8$ ➔ $0 = (x - 4)(x + 2)$
So x = 4, integers are 4, 5, 6: sum = 15. Pass: $\frac{B}{X-3} = \frac{15}{-2-3} = -3$. **Ans. B = 15, Pass: -3**

Yellow Relay Seat C

BG = 3 and GD = 12. Thus $AG^2 = 36$, AG = 6, so AC = 12. Pass: $(X + 1)^{C/2} = (-3 + 1)^{12/2}$
$(-2)^6 = 64$. **Ans. C = 12, Pass: 64**

Yellow Relay Seat D

(1) x + 3y = 1, (2) $y = x^2 + x - 1$. In (1): x = 1 – 3y. Subbing into (2):
$y = (1 - 3y)^2 + (1 - 3y) - 1$ ➔ $y = 1 - 6y + 9y^2 - 3y$ ➔ $0 = 9y^2 - 10y + 1$ ➔
0 = (9y – 1)(y – 1), y = 1 or 1/9. In (10): If y = 1, x = -2. If y = 1/9, x = 2/3. Adding all four: 1 + (-2) + 1/9 + 2/3 = -1 + 7/9 = -2/9. Pass: $\frac{-X}{3D} = \frac{-64}{3(-2/9)} = \frac{-64}{-2/3} = 64(3/2) = 96$.

Ans. D = -2/9, Pass: 96

Yellow Relay Seat E

From Pink E: (a + b) – (h + k) = (8 + 10) – (5 – 3) = 16. Pass: X/(E-4) = 96/(16 – 4) = 8

Ans. E = 16, Pass: 8

Answer Sheet States 2013

Individuals Round 1

1. $y = -2x - 11$
2. 97
3. 5/12

Individuals Round 2

1. 8
2. $60 - 32i$
3. 5

Individuals Round 3

1. 192
2. 24
3. 226

Individuals Round 4

1. ½
2. $36\sqrt{2}$
3. 3

Individuals Round 5

1. 135^o
2. $\frac{5}{2}x^3y^6$
3. 500

Individuals Round 6

1. $55^o, 55^o$ or $70^o, 40^o$
2. -79
3. $0^o, 30^o$ and 150^o

Team Round 1

1. $3\frac{1}{3}$
2. $45
3. $\frac{x-2}{x+2}$
4. 235_6
5. $-1 < x < 0$ or $1 < x < 1\frac{1}{2}$
6. $(-\frac{1}{2}, 0), (5\frac{1}{2}, 0)$
7. $8\sqrt{3} + 24$
8. $2\sqrt{3}$

Team Round 2

1. 6
2. 10,155
3. $\frac{x^2 - x + 1}{x^2}$
4. 24
5. $y = -7x + 5$
6. 5
7. $x \le \frac{-3-\sqrt{17}}{2}$ or $0 \le x \le \frac{-3+\sqrt{17}}{2}$ or $x \ge 3$
8. 18

Relays

	Blue		Green		Pink		Yellow	
	Answer	Pass	Answer	Pass	Answer	Pass	Answer	Pass
A	2	16	-4	1	4	-4	5	-2
B	15	8	36	9	12	-2	15	-3
C	64	16	192	12	8	16	12	64
D	2	4	1	11	-7/16	-7	-2/9	96
E	12	28	6	3	20	10	16	8